CONTRIBUTION OF MUSLIM SPAIN TO MATHEMATICS

DISSERTATION

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BY

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This is to certify that Mr. Adam Malik Khan has completed his M.Phil. Dissertaion on Contribution of Muslim spain to Mathematics under my supervision, and that the work is his own original contribution and suitable for sumbmission for the award of the degree of M.Phil..

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Adam Malik Khan

(ADAM MALIK KHAN)

P R E F A C E

The present work entitled "The Contribution of Muslim Spain to Mathematics" is a sincere attempt to collect together all the possible extant material concerning the Muslim mathematicians of Spain whether they were Spanish by birth or were migrants who settled there. Since no complete work of such nature exists, therefore, this is just a first attempt to begin work of this kind.

Our presentation of this work is as best as could be done. It is not claimed that all the names that matter have necessarily been included or discovered. Therefore, there may be some deficiencies.

The dissertation is finally divided into four chapters apart from the introduction and conclusions. The first chapter contains the developments of early civilization, nations and schools. In the second chapter Spanish-Muslim mathematicians of the 10th, 11th and 12th centuries A.D. alongwith their works are discussed. Similarly the third chapter deals with the Spanish-Muslim mathematicians and their works during the 13th, 14th and 15th centuries. Apart from those mathematicians discussed in the second and third chapters there were several others about whom not enough material could be obtained. They are included and discussed in a separate chapter - the fourth.

System of Transliteration of Arabic Characters

Consonants				Long Vowels		Diph thongs	
ء	' Except when initial			آ	ā	وْ	aw
ب	b	ط	ṭ	و	ū	ـَيْ	ay
ت	t	ظ	ẓ	ى	ī	ـِيَّة	iyy
ث	th	ع	'	Short Vowels		ـُوّ	uww
ج	j	غ	gh	ـَ	a	ة	a; a
ح	ḥ	ف	f	ـُ	u	ال	al- ,
خ	kh	ق	q	ـِ	i		
د	d	ك	k				
ذ	dh	ل	l				
ر	r	م	m				
ز	z	ن	n				
س	s	و	w				
ش	sh	ه	h				
ص	ṣ	ى	y				
ض	ḍ						

INTRODUCTION

The Muslims of the early period had great interest in almost all the fields of learning. They made remarkable contributions in every field of knowledge under the Umayyads and the Abbasids. The Muslims of Spain in similar fashion carved out for themselves a place in their contribution to knowledge at par with their brethren in the other parts of the Islamic world.

Mathematics was an important subject which was studied and developed by them alongwith other subjects of religious as well as natural sciences such as history, geography, Quran, hadīth, fiqh, tafsīr, medicine, pharmacy, gardening, astronomy, astrology, music, alchemy, surgery etc. For quite a long period Spain remained an important seat of learning. Mathematics was such a subject which became essential for the Muslims soon after the emergence of Islam for the performance of many important religious duties. Mushtāq and Tan remark that:[1]

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1. Mushtāq and Tan, Mathematics:The Islamic Legacy, pp. 17-18.

> Besides the performance of some of the fundamental duties of Islamic faith was near to impossible without scientific knowledge, and in particular, knowledge of Mathematics. Fasting, pilgrimage and numerous other religious ceremonies required careful and scientific understanding of the lunar calander... Zakat, the laws of inheritance and waqf (religious endowments) are purely in need of mathematics. Without the knowledge of mathematics one can not reach to a fixed result regarding these religious duties.

Therefore, Islam from its very beginning teaches its believers to seek knowledge. The first word of the revelation Iqra' (read) is a great pointer towards seeking of knowledge. Muslims are encouraged by Allāh and Prophet Muḥammad (peace be upon him) through Quran and hadīth for seeking knowledge. As the Quran says, "Allāh will exalt those who believe and those who are given knowledge to high degrees" (LVIII:11). Their attention are also diverted towards the study of sky and earth to find proof to their faith. Similarly it is said, "Whoever is given knowledge is indeed given abundant wealth" (II:269).

The importance of knowledge may be recognized from the fact that it occurs at least 750 times in the Quran and is indicated at numerous places directly or indirectly through various words like 'understanding','remembering', 'thinking', 'reasoning', etc.

The importance of knowledge is also proved by the tradition of the prophet Muḥammad (p.b.u.h.). According to various traditions, he himself prayed for knowledge as "My Lord, increase me in knowledge". He directed the believers to "seek knowledge from the cradle to the grave, no matter if their search took them as far afield as China". The status of knowledge was so much high ranked that it was told "He who travel in search of knowledge travel along God's path of paradise", "the ink of scholars is worth more than the blood of martyrs" and "to listen to instructions of science and learning for one hour is more meritorious than attending the funerals of a thousand martyrs - more meritorious than standing up in prayer for a thousand nights". Muslims were bound to seek knowledge as the prophet Muḥammad (p.b.u.h.) said, "Seeking of knowledge is obligatory upon every Muslim". Many other tradition of the prophet Muḥammad (p.u.b.h.) emphasised on getting and teaching knowledge. In Bukhārī it is mentioned that the downfall of a nation is sure if it gives up the acquisition of knowledge.

Therefore to follow Quran and the sayings of the prophet Muḥammad (p.b.u.h), Muslims required knowledge through various means. A number of Muslim scholars performed a marvellous job either independently or under patronage of the rulers. Throughout the four centuries (8th to 11th century A.D.) or even more the scientific learning by the Muslims reached its height as and when they first translated the scientific works obtained from various parts of the world, wrote commentaries and summaries and then developed them.

Mathematics was the first priority of Muslim scholars alongwith other religious sciences. Arithmetic, geometry, plane and spherical trigonometry and algebra were the branches of mathematics on which Muslims of Spain paid their attention. They worked and developed almost all these branches to their height.

Mathematics, as mentioned in The New Encyclopaedia Britannica:[2]

> ...Is a calculational science that characterizes the automated form of reasoning process and the methods of thought that rely upon such things as physical models and number storage and manipulation. The end results of a mathematical calculation may be a statement, a decision, a number, a geometrical drawing, a plan, or other recorded conclusion.

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2. The New Encyclopaedia Britannica, (Fifteenth Edition). Vol. 11.

Arithmetic is supposed to be the branch of mathematics which was studied by man before all learnings and even before written language came into existence which seems to be obvious because he, first of all was needed to count when he saw different objects, required to add one to the other, subtract and divide them into different classes. Arithmetic, therefore, 'is the concept of numbers and operations on numbers'.

Geometry, as the word itself shows, is the science of earth measurement ('geo' means 'earth' and 'metre' means 'measurement'). So according to A.A. al-Daffa', "Geometry is a science which not only leads to the study of the properties of space, but also deals with the measurement of magnitude".[3] According to its need, this science has length, width and height as its three dimensions which help in measurement, especially in the measurement of earth.

Similarly trigonometry is another branch of mathematics which is 'the science of angles'. As is obvious from the word 'trigonometry', it has its three parts: 'tri', 'gonon', and 'metria' which have the

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3. 'Alī 'Abdullah al-Daffa', The Muslim Contribution to Mathematics, p. 82.

meanings 'three', 'angle', and 'measurement' respectively. This branch of mathematics may also be regarded to deal with the measurement and calculation of the sides and angles of a triangle.

Algebra is 'the science of generalization arithmetic'. It may also be defined as "that branch of mathematical analysis which reasons about quantities using letters to symbolize them". For example if x is any number then $x+x+x+x+x = 5x$

Muslim Spain also produced a number of Muslim mathematicians who started, their activities in this field from almost the middle of the 10th century A.D. and continued their endeavours till the fall of Muslim rule in the country in 1492 A.D.The exact number of mathematicians produced by Spain and their contributions may never be known as a huge material of historical importance was destroyed by the Christian rulers of Spain who succeeded the Muslims.

CHAPTER - I

EARLY DEVELOPMENT OF MATHEMATICAL SCIENCE

As soon as man came into existence, he started applying his mind to create something new. He developed the sense of counting which led the concept of numbers, the history of which, therefore, runs through a considerable period in the history of man. The task of tracing the gradual development of this fantastic discovery of man's civilization is immensely difficult because the ability of counting is certainly a gifted phenomenon to all living beings and that is why man, animals and birds naturally formed groups, as sedgwick and Tyler have rightly opined:[1]

> The periods at which primitive man of different races began to have conscious appreciation of the phenomena of nature, of number, magnitude and geometric forms, can never be known nor the time at which their elementary notions began to be so classified and associated as to deserve the name of science very early in any civilization, however, there must obviously have been developed simple processes of

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1. W.T. Sedgwick and H.W. Tyler, A Short History of Science, Revised by H.W. Tyler and R.P. Bigelow, pp. 15-16.

> counting and adding of time and distance measurement, of the geometry and arithmetic involved in land measurement and in architectural design and construction.

It is universally accepted that mathematical science seems to have first assumed definite form in Greece. India and China are, nevertheless, also supposed to have mathematical thinking further back than any record runs but their civilizations made minor, though fundamental contributions at a much later stage. Historians, like H.V. Hilprecht[2], D.E. Smith[3], and P. Tannery[4] have established, beyond doubt that Babylonia and Egypt, infact, were the prime civilization who exerted a determining influence on general evolution of various sciences, particularly that of mathematical science.

Though there are evidences that Babylonians and Egyptians exchanged ideas with each other and, in subsequent span of time (rather after very many

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2. Cf: H.V. Hilprecht, 1906, Mathematical, Metrological and Chronological tables from the Temple Library of Nippur, University of Pennsylvania, Vol. XX, part I series (A) by F. Cajori, , A History of Mathematics, p. 7.

3. Cf: D.E. Smith, 1907, Bulletin of American Mathematical Society, Vol. 13, p. 392: by F. Cajori, Op. cit.

4. Cf: Tannery, Revue Philosophique, Vol. I, 1976, p. 170, Vol. XIII, 1881, p. 210, Vol.XV, 1883, p. 573 by F. Cajori op. cit.

centuries), furnished Greece with a certain nucleus of mathematical knowledge. Different civilizations, communities and nations invented different type of notations for the numbers. For example, the Babylonians used  for 1,  for 2, similarly  or 3  for 4,  for 10,  for 23,  for 30,  for 100,  for 1000 and similarly  for 2000.[5] The Egyptians used  as 1,  as 10,  as 100,  as 1000,  as 10,000  as 1,00,000  as 1,000,000,  as 10,000,000. Thus 23 was written as .[6]

The period of Babylonian civilization is generally known from 3100 to 2100 B.C. In the period of Sargon[7] its people was much interested in mathematics which is proved from the business of its people. A record of eclipse of his period is reported to be found which shows their far advancement in numeral system.[8]

It has been found that the Babylonians made two tablets. The first gives the numbers in square[9] upto

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5. F. Cajori, op. cit., p.4.

6. Ibid., p. 11.

7. A Great Ruler of Babylonia, Flourished about 2700 B.C.

8. D.E. Smith, History of Mathematics, Vol. I, p. 36.

9. When a number of multiplied with itself then it the result is known as the square of the number.

60^2. The method they had adopted to mention it, is the same as is used today (1, 4, 9, 16, 25, 36 and 49). They then give 8^2 = 1.4 or (60+4) where 1 is used for 60 and point (.) for addition (i.e. for (+)), 9^2 = 1.21 (60+21), 10^2 = 1.40. Similarly, they used 2 for 120 (2x60). In this way the table upto sixty is completed.

The second tablet gives the illuminated portion of moon's disc of every day from new to full moon. It divides the whole disc into 240 parts. It gives first five days of illumination in geometrical progression (5, 10, 20, 40, 1.20[10]) and the next upto 15th in arithmetic progression (1.20, 1.36, 1.52, 2.8, 2.24, 2.40, 2.56, 3.12, 3.28, 3.44, 4) having a difference of 16. A symbol for zero, ⪡ which is found in their works shows that they were acquainted with the number but, Cajori says that it was never used in their calculation.[11] These works of Babylonians were used by the Greeks. Ptolemy used the sexagesimal fraction in his almagest and to represent blanks in the sexagesimal numbers he used the omincron 'o', but not as zero. For zero they had some other symbol. The

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10. As mentioned on page No. 1.20 = 1x60+20 = 80.

11. F. Cajori, op. cit., p.5.

Greek scholars such as Hipparchus, Hypsicles and Ptolemy borrowed and introduced the sexagesimal fractions (as designated 1/2 and 1/3 by 30 and 20) into Greece. These sexagesimal fractions may have been introduced into India with the use of zero and they were used in mathematical and astronomical calculations until the sixteenth century unless the decimal fractions took their place. They also knew the methods of multiplication, division, square roots and computation, but in geometry their knowledge was limited upto the areas of squares[13], rectangles[14], right-triangles[14] and Trapezoids[15].

The Egyptians :

With the reference of C.A. Bretschneider's Die Geometrie und die Geometer vor Euklides, Cajori mentions the words of Aristotle as:[16]

> The mathematics had its birth in Egypt, because there the priestly class had the leisure needful for the study of it. Geometry in particular is said by Herodotus, Diodorus, Diogenes, Laertius, Iamblichus and other ancient writers to have originated in Egypt.

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12. A four equal sided figure having all four angles of 90° each.
13. A parallelogram having equal angles.
14. A triangle with one angle as 90°
15. A quadrilateral with one parallel pair and one non-parallel pair of sides.
16. Ibid.,p.9.

In the practical geometry the Egyptians were far advance as they divided the whole land of Egypt into equal quadrangles[17] to distribute each to every countryman. For the purpose of measuring the land they used a unit 'khet', for 1 khet = 1.66 m or its thrice, using which they found out the area of an isoceles triangle [18] as 'half the product of the base and one side', the area of an isosceles trapezoid as 'multiplying half the sum of the parallel sides by one of the non-parallel sides' and in finding the area of a circule they deducted 1/9 from the length of diameter and squared the remainder. The value of π[19] they used was $(16/9)^2$ or 3.1604.

In calculations, the Egyptians used to move their hand from right to left with pebbles while another 'Hellenes' move it from left to right. As early as 2000 B.C. they knew about the construction of right triangle. While writing fractions, different civilizations used to keep numerator or denomintor as constant: the Babylonians kept 60 as its denominator while 60 was replaced by the Romans by 12. In the same

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17. Plane four sided figure.

18. A figure surrounded by three sides, two of which are equal.

19. It is denoted for the ratio of circumference of a a circle to its diameter.

way numerator was kept constant by the Egyptians and the Greeks. The Ahmes papyrus[20] used fractions that have the unit numerator. The Egyptians were acquainted with arithmetic progression as well as geometric progression and quadratic equation[21] in algebra. They used the symbol ' $\sqrt{\ }$ ' to represent square root. In Egyptian arithmetic it seems to be a lack of simple comprehensive symbolism which the Greeks could not remove. Their knowledge in geometry is remarkable and doubtless. Many great scholars of Greece received the knowledge of geometry by visiting Egyptian priests, but in the next two thousand years they had made no progress. They invented the formula 2/2n+1 to reduce fractions[22] to a sum each of whose numerators is unity (e.g. 1/24 + 1/58 + 1/174 + 1/232 = 2/29 and 1/56 + 1/679 + 1/776 = 1/97). They used to repeat additions in multiplication and subtractions in division. In Egyptian arithmetical symbolism, two legs walking foreward represents the addition of numbers and walking in backward direction or a flight of arrows shows the symbol for subtraction

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20. Written by Ahmose between 1700 and 2000 B.C., is the most important source of information of that period.

21. An equation of degree two.

22. A number of the form p/q where p and q are natural numbers.

where as the sign $\nless$ gives the indication of equality. Ahmes gives the area of a circular field by $(d - 1/9)^2$, where d is used for the length of diameter of the circle.

The Greeks :

Greeks did an excellent job in the contribution of mathematics especially in geometry while the Hindus are regarded as the great contributors in the field of arithmetic. The Greeks are reported to get little or more knowledge from the Egyptians. Many early Greek Scholars such as Thales, Pythagoras, Cenopides, Plato, Democritus and Eudoxus visited and received knowledge from the Egyptian priests. "Whatever we Greeks receive we improve and perfect", says Plato.[23] Many schools were founded under different circumstances in Greeks.

The Ionian School :

The first among them is Ionian School, founded by Thales (640 - 546 B.C.) of Miletus who is supposed to be one of the 'seven wisemen'.[24] He is the man who

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23. Morris Kline, Mathematics in Western Culture, p. 24.

24. F. Cajori, op. cit., p. 15.

introduced geometry into Greece. He is said to have invented many theorems in geometry such as on the equality of vetical angles, the equality of the angles at the base of an isosceles triangle, the bisection of a circle by any diameter and two triangles are congruent if a side and two adjacent angles are equal. He used the theorem on similar triangles. All the angles inscribed in a semi-circle[25] are right angles, and the sum of all three angles of a triangle is equal to the sum of two right angles, are described by him. This school produced many great mathematicians of the time such as Anaximenes (b. 570 B.C.), Mamercus, Mandrytus and Anaximander (611-545 B.C.). Anaxagoras (500-428 B.C.) worked for the first time on the quadrature of the circle.

The Pythagorian School :

This school was founded by Pythagoras (569-500 B.C.) at Croton in 532 B.C. Pythagoras is one of the greatest mathematicians in the history of mathematics to whom a famous theorem of geometry 'The phythagoras theorem'is ascribed which looks to be doubtful because the Egyptians were also familiar with it. According to

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25. It is the region of the circle bounded by its diameter.

this theorem "In a right angled trianle, the square of the length of its hypotenuse is equal to the sum of the squares of the lengths of its other two sides". He was born at Samos. It is a great matter of interest that for any particular invention it is difficult to say that it is ascribed to some particular person . They used to refer all their discoveries to the founder of the school which was of course as a result of their unity as a body of brotherhood and discipline. According to him a plane may be divided up into any kind of figures. The pythagorians were familiar with the circle[26], sphere[27] and polygon[28] and also with the duplication of cube[29] and the generation of cones[30] and cylinders[31]. The Pythagorians discovered the hypotenuse and other two sides of a right angled triangle as $(2n^2 + 2n + 1)$, $(2n^2 + 2n)$, $(2n + 1)$ respectively.[32] The school existed for two centuries.

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26. Space enclosed by a curved line every point of which is at the same distance from the centre.

27. The locus of a point in space which moves such that its distance from a fixed point is constant.

28. A figure bounded by more than four straight lines.

29. A solid figure bounded by six square faces.

30. A solid figure obtained by revolution of a straight line about its one end.

31. A solid figure obtained by revolution of a rectangle about its one side.

32. W.W. Rouse Ball, A Short Account of the History of Mathematics (5th ed.). p.26

Its founder was murdered in Metapontum due to their involvement in politics. In the later period the Pythagorians exercised great influence on the study of mathematics at Athens.

The Sophist School :

After the fall of Pythagorian school this school came into existence. The word 'Sophist' means 'wise-men' and they were the teachers belonging to Pythagorian school of Sicily. They came to Athens for the purpose of teaching on its people's demand. They paid their special attention to the solution of three problems and in this connection they made several discoveries:

1. To trisect an arc or an angle;
2. To double the cube or to find a cube whose volume is double that of a cube; and
3. To make a circle, square or any rectilinear figure having the area equal to the area of a circle.

Philolaus, a Pythagorian of later period and a teacher of this school was such an important mathematician whose works were bought by Plato, the founder of platonic school.

The Platonic School :

This school was founded by Plato (429-348 B.C.) at Athens about 398 B.C. He visited Cyrene and became a pupil of Theodorus there, then went to Egypt, Lower Italy and Sicily for the sake of knowledge. Plato was so much interested in the field of mathematics that he did not like any body to enter his house who had no knowledge in geometry. He defined a 'point' as 'the beginning of a line' or as 'an indivisible line' or 'a line as length without bredth', and described the point, line and surface as the boundaries of line, surface and solid respectively.[33] A great invention of Plato is the 'analysis' as a method of proof, which was also used by Hippocrates. He solved the problem of duplication of cube and worked on pyramid[34], cylinder and cone. By cutting three kinds of cones: the right angled; the acute angled[35]; and obtuse angled[36]; Menaechmus, an associate of plato got three sections,

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33. Cajori, op. cit., p. 26.

34. Structure with triangular sides meeting at the top.

35. An angle whose measure lies between 0^{o} and 90^{o}.

36. An angle whose measure lies between 90^{o} and 180^{o}

the parabola[37], ellipse[38] and the hyperbola[39]. Dinostrus, Edoxus, Theaetetus, Leodamas and Neocleides were among his pupils who worked on mathematics and contributed the field by their researches, developments and creative writings.

The Athenian School :

Aristotle (384-322 B.C.) was a scholar of the Athenian school. He worked on parallelogram[40]. He also improved some difficult definitions of geometry. The theorem, 'The sum of the exterior angles of a plane polygon is four right angles is attributed to him.[41]

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37. The locus of a point which moves such that its distance from a fixed point is equal to its distance from a fixed straight line.

38. The locus of a point which moves such that the ratio of its distance from a fixed point to its distance from a fixed straight line is constant which is less than unity (i.e. 1).

39. In the case of hyperbola the constant is greater than unity.

40. A quadrilateral with its opposite sides as parallel.

41. W.T. Sedgwick and H.W. Tyler, op. cit., p. 89.

The First Alexandrian School :

After having wandered at many centres of learning like the Ionian land, lower Italy and finally Athens, geometry returned to its birth place, Egypt. It was due to the consequences of the defeat of Athens with the hands of Philip of Macedon, the conquest of the world by Alexander the Great and the establishment of a small Kingdom in Egypt by Ptolemy, son of Alexander after the break of Alexander's kingdom into pieces. Ptolemy, the new ruler of Egypt founded a school about 306 B.C. at Alexandria which was called 'the First Alexandrian School'. Inspite of having the responsibilities of a king he used to devote a considerable time to scholarly activities. He opened a University. Many foreign scholars were invited to hold different posts in the University. Demetrius Phalereus took the charge of library and Euclid holded the charge as the head of mathematical department. In the first century three greatest mathematicians of antiquity Euclid, Archimedes and Appolonius belonged to the First Alexandrian School. Euclid wrote his famous book Elements in 13 volumes on geometry together with the elementary arithmetic and algebra. In response of this book he got the title 'The author of the

Elements'. Even today his Elements is regarded as the best introduction to the mathematical science. Gow says:[42]

> Not much later than these [of the academy] is Euclid who wrote the Elements, arranged much of Eudoxus's work, completed much of Theaetetus's, and brought to irrefragable proof propositions which had been less strictly proved by his predecessors(Greek Math P.137).

The treatises on porisms [a special type of geometrical proposition], the 'Data', containing geometrical theorems, on Fallacies to save from false reasoning, division of figures, surface-Loci, conic sections and the works on phenomena on optics and on catoptrics[43] which deals with application of geometry, are all ascribed to Euclid. Archimedes (287-212 B.C.) is another mathematician of the Alexandrian school who is thought to have studied thoroughly all previous works on mathematics. Nine works are ascribed to him which are on centres of plane gravities, the quadrature of the parabola, sphere, cylinder, the measurement of the circle, Spirals, conoids and spheroids, floating

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42. W.T. Sedgwick and H.W. Tyler, op. cit., p.104.

43. It deals with the reflection in Plane, concave and convex mirror.

bodies, the sand counter and fifteen Lemmas.[44] Appolonius of Perga (260 - 200 B.C.) is supposed to be next to Euclid and Archimedes who got the title 'Great geometer' for his brilliancy in the field. He wrote his great work on conic section in eight books (the eighth of which is lost), having four hundred propositions. His work is divided into two classes: on the quadrature of the curvilinear figures, which gave birth to the calculus and which were perfected by Kepler, cavalieri, Fermat, Leibnitz and Newton; and the theory of conic sections. The other mathematicians of this school were cannon, Dositheus, Zeuxippus, Diocles, Perseus, Zenodorus, Hypsicles, Hipparchus (the astronomer) who is supposed to be the originator of trigonometry, Heron the Elder of Alexandria and Geminus of Rhodes who worked on different sections of geometry.

With the conquest of Egypt by the Romans in the first century B.C. the scholars of the East and the West began to meet at a large scale and the mutual transformation of knowledge started. This new system was called by the scholars as 'neo-pythagoreanism' and 'neo-platonism', and with this period which is said to begin with

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44. F. Cajori, op. cit., p. 34.

the Christian era, a new study 'the theory of numbers' started. The famous mathematicians of this school were: Claudius Ptolemaeus who wrote his famous work Syntaxis Mathematica (or according to the Arabs, Almagest) on astronomy in 13 books in which he discussed trigonometry also which is based on the works of Hipparchus; Serenus of Antinocia who wrote two books on cone and cylinder; spherica of Menelaus of Alexandria, written on accuracy of spherical triangles[45] and circle; Posidonius' definition on parallel lines that are coplanar and equidistant; several books especially a collection of mathematical papers in eight books of Pappus of Alexandria (b. about 340 A.D.) which is supposed to be a synopsis of Greek mathematics;[46] a book on arithmetic of Theon of Alexandria written for the facility of the readers to study the Plato's writings; the works of Diophantus who is the writer of short essay on polygonal numbers, a treatise on algebra and a work on porisms; Nicomachus' work who published a work on arithmetic, having the

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45. It is defined as the triangle formed by three spherical arcs or curved lines.

46. W.W. Rouse Ball, op. cit., p. 99.

object to study the properties of numbers and their ratios, which also contains the even [47], odd [48], prime[49] and perfect[50] numbers and discusses fractions, polygonal and solid numbers and treated with ratio proportion and the progression; and lastly Iamblichus' (Circ 350) work who worked on the properties of numbers.

This may be said, keeping their work in mind that during the last 500 years no more creative work was done by the Greeks though many commentaries were written by them. Mentioning the characteristics of ancient geometry Cajori states it as, 'A wonderful clearness and definiteness of its concepts and an almost perfect logical rigor of its conclusions' and 'a complete want of general principles and methods'.[51]

Since there was no perfect system of counting among the Greeks in those days, they used their fingers

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47. An intiger (± 1, ± 2, ± 3,.....) which is divisible by 2.

48. An intiger which is not divisible by 2.

49. A natural number (1,2,3,.....), greater than 1 which is divisible by 1 and itself only.

50. A number which is equal to the sum of its factors.

51. F. Cajori, op. cit., p. 51.

or pebbles for the purpose of counting in different ways. But later, Pythagoras introduced abacus[52] into Greece which may be imported perhaps from China where it was used in the name 'Swan Pan', after having visited to Egypt and India.

Like the Babylonians and Egyptians the Greeks also had their own numbers which are as follows:

α	β	γ	δ	ε	ς	ζ	η	θ	ι	κ	λ	μ	ν
1	2	3	4	5	6	7	8	9	10	20	30	40	50

ξ	ο	π	ϙ	ρ	σ	τ	υ	φ
60	70	80	90	100	200	300	400	500

χ	ψ	ω	ϡ	͵α	͵β	͵γ
600	700	800	900	1000	2000	3000

M	$\overset{\beta}{M}$	$\overset{\gamma}{M}$
10000	20000	30000...

In the Greek numbers there was no concept of zero. They used the term 'arithmetica' and 'logistica' for the science of numbers and art of calculation and expressed the earlier as the 'theory' and later as the 'practice'. The method of writing fractions among Greeks

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52. An abacus is a frame having many rows of string tied with its boundaries and the string has moveable balls in it.

was something different. When they needed to write 59/88, they expressed in their own manner as εθ'η''η''ηη'' (The numerator was first written with one accent and then the denominator in twice with two accent). When the numerator is unity then it is removed and only denominator is written with two accent. Archimedes was familiar with square roots which he gave in his <u>Mensuration of the Circle</u>. He claimed that the sand grains of the earth and even of the universe may be counted.[53] The Pythagorians are credited to differentiate between even and odd numbers and proved various new things in connection with these numbers. To acknowledge the arithmetic, geometric and harmonic progressions[54] they discovered formulae.

<u>The Romans</u>:

The Romans were not much more interested in the study of mathematics. That is why in their schools very elementary arithmetic (calculation with the help of abacus), a few practical rule in geometry, but a high level of surveying were taught. They continued the method of counting on fingers and even developed the

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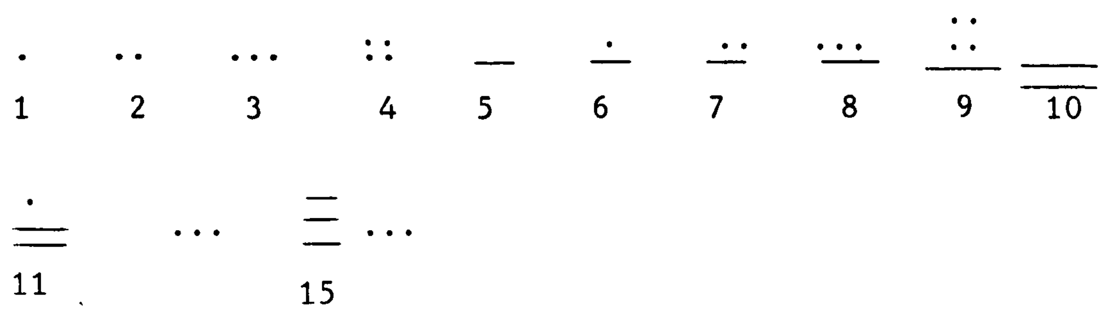
53. F. Cajori, <u>op. cit.</u> , p.54.

54. Three numbers a, b and c are said to be in harmonic progression if a/c = a-b/b-c

counting in this way to 10000 or more.[55] A Roman mathematician, Boethius wrote Institutis Arithmetica which is a translation of the arithmetic of Nicomachus and also several books on geometry. Another mathematician, cassiodorius was the first to use the terms 'rational' and 'irrational' in the sense of arithmetic and algebra respectively.

The Mayas :

Maya race flourished in the Central America and Southern Maxico. Their development in mathematics began with the beginning of first century A.D. and got a higher rank in the intellectual achievements. They were familiar with the vigesimal number system with the notation of zero and the principle of local value five or six centuries before the Hindu introduced their decimal number system with zero and principle of local value. The following are the symbols of the numbers used by Mayas:

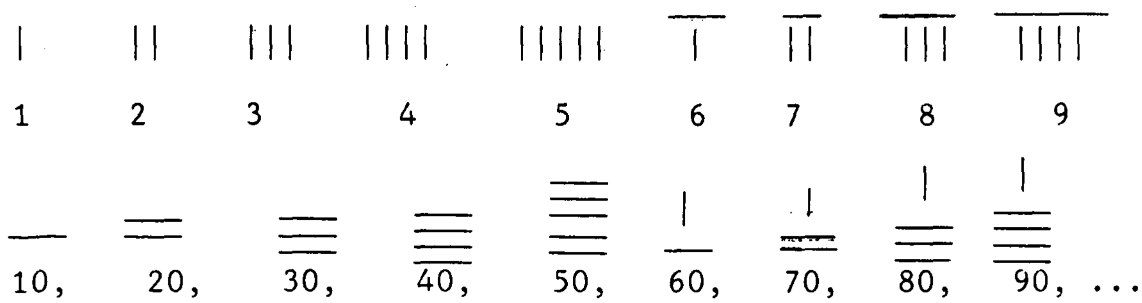


55. W.W. Rouse Ball, op. cit., p. 113.

It shows that the dot was used for 1 and bar for 5 where as 'zero' was represented by 'a half closed eye'. They also used the terms 'Kins' as days, 'uinal' was used for 20 days. They supposed 18 uinals as one 'turn' which ultimately equals to 360 days. 29 tuns were fixed for one 'Katun' = 7200 days, 20 Katun for one 'cycle' = 144,000 days and 20 cycles for 1 'great cycle' = 2,880,000 days. From this supposition of symbols and their usage it is quite obvious that the Mayas had the numbers upto 2,880,000 or more.

The Chinese :

In China, it came to be known through different sources that their mathematical study is also very old. This may be proved from the fact that its oldest, lost publication' Chou-Pei is of 1100 B.C. Another Chinese great work on arithmetic is Chiu Chang Suan Shu (Arithmetic in nine sections) contains the knowledge of mensuration; gives the areas of triangle (1/2(b x h)}, Trapezoid (1/2 (b+b')xh), of circle (1/2 C . 1/2 d or 1/4 Cd or 3/4 d^2 or 1/12 C^2, where C is the circumference and d is the diameter of the circle); fraction; commercial arithmetic including percentage and proportion;partnership and square root as well as cube root of numbers.[56] It also deals with the multiplication and division of fractions, rules for finding the volumes of prism, cylinder, pyramid, truncated pyramid and cone, tetrahedron[57] and Wedge[58]. The symbols of positive and negative numbers and many mathematical problems are given in this book. Sun-Tsu wrote a book Sun Tsu Suan-ching (arithmetical classic of Sun-Tsu) in which he mentions the symbols for the numbers:



Then again vertical is used for hundreds and horizontal for thousands. In this way 6619 is written as

Lue Hui wrote Hai-Tao Suan-Ching (sea Island arithmetical classic) on algebraic manipulation. Yang

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56. F. Cajori, op. cit., p. 71.

57. Solid figure in four plane sides.

58. V-shaped piece.

Hui flourished in 13th century and worked on the summation of arithmetical progressions of the series $1 + 3 + 6 + 10 + \ldots + (1+2+3+\ldots+n)$ and proved that it is equal to $n(n+1)(n+2)/6$. He also proved that the sum of the squares of the series of natural numbers upto n, $(1^2 + 2^2 + 3^2 + \ldots + n^2) = (1/3)n(n+1/2)(n+1)$. Many other algebraic works as linear equation[59] as well as quadratic and quartic equations upon which he did considerable work, are also ascribed to him. The other works by the Chinese mathematicians are: <u>Suan-hsiao Chi-meng</u> which is an introduction to the mathematical studies; <u>Szu-Yuen Yu-Chien</u> (The precious Mirror of the four Elements) in the last decade of thirteenth century, written by Chu Shih-Chieh; and <u>Suan-fa T'ung-tsung</u> (A systemised Treatise on arithmetic) by Cheng Tai-Wei having the diagram of abascus and expressing the method of its use; on the solution of numerical equations, Chin Chiu-Shao wrote <u>Su-Shu Chiu-Chang</u> (Nine sections of Mathematics); LI Yeh devoted his writings on circle and wrote <u>T'Se-Yuan Hai-Ching</u> (sea mirror of the circle measurements) in which he gave the symbol 'o' for zero

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59. It is an equation the degree of which is one.

and presented the negative number also as he gave $\perp 0$ = 60 and $\cancel{\perp}0$ = - 60. After the 13th century the downfall began in chinese mathematics and then no great work was prepared by them, but it is remarkable that the magic squares and magic circles were originated there.

The Hindus :

One of the high ranked civilization in the field of mathematics especially arithmetic is Indian or Hindu. Though only a particular caste, the Brahmins were involved in the study, but they made a marvellous contribution in the field of mathematics. It is said that where Greeks worked on geometry, alongwith the study of mathematics they used to get the knowledge of astronomy and almost all of them got expertised in astronomy, the Hindus contributed more in arithmetic. The Indian mathematics may, some more or less be influenced in response of their commercial relations with Alexandria, a great centre of learning of mathematics for a long time. Its scholars worked on numbers, numerical symbolism and algebra more devotionally than on geometry and trigonometry. Their geometry was mainly based on mansuration, and trigonometry on arithmetic more than on geometry.[60] Many great mathematicians like Aryabhata, Brahmagupta, Mahavira, Sridhara, Padamanabha, Bhaskara and Varaha Mihira were famous for their works who devoted their lives to the study of mathematics and astonomy. A famous astronomical work, Surya Siddhanta (knowledge from the sun) was written in the fifth century A.D. on which varaha Mihira wrote a summery entitled Pancha Siddhantika, a portion of which was devoted to mathematics. Similarly Aryabhatiya was a work on astronomy, written by Aryabhata (b. 476 A.D. at Patliputra) having a section on mathematics. Brahmagupta was also a contemporary of Aryabhata, born in 598 A.D. and wrote his famous work Brahma Sphuta-Siddhanta (the revised system of Brahma). He devoted two of its chapters to arithmetic, algebra and geometry. Mahavira wrote Ganita Sara-Sangraha on Hindu geometry and arithmetic. Sridhara wrote Ganita Sara (Quintessence of calculation). Padmanabha had a knowledge of algebra and wrote on this. Bhaskara is the name without which the history of Hindu mathematics may not be completed. He was born in 1114 A.D. He is supposed to be a higher ranked mathematician than

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60. F. Cajori, op. cit., p. 83.

Brahmagupta. Siddhanta Siromany (Diadam of an Astronomical system) is his work which contains two chapters: Lilavati on arithmetic and Bija Ganita on algebra. Mentioning the invention of Arabic notation by the Hindus, Cajori says:[61]

> The grandest achievement of the Hindus and the one which, of all mathematical inventions, has contributed most to the progress of intelligence, is the perfecting of the socalled Arabic notation. That this notation did not originate with the Arabs is now admitted by every one. Until recently the preponderence of authority favoured [Sic] the hypothesis that our numeral system, with its concept of local value and our symbol for zero, was wholly of Hindu origin.

The Hindus are credited to be familiar with interest, discount, partnership, alligation, summation of arithmetical and geometrical series as well as permutation and combination. The arithmetic which is being used even today, either the form or the spirit is ofcourse of Hindus.

The Arabs :

As soon as Islam spread and Muslims conquered new territories and settled there, they had to suffer

61. F. Cajori, op. cit., p. 88.

with new kinds of deseases which they knew nothing about. For the purpose of treatment they had to meet Greek and Jew doctors, as the medical knowledge was confined to them although some Greek schools were running in the Arab provinces of Egypt and Syria i.e. at Damascus, Emesa, Edessa and Antioch. Arnold, regarding the Arab contribution to mathematics says, "... The Arabs are before all else the pupils of the Greeks; their science is a continuation of Greek science which it preserves, cultivates and on a number of important points developes and perfects".[62] The translations of Greek medical works especially of Hippocrates, Aristotle and Galen were started into Arabic near about 800 A.D. by the order of Hārūn al-Rashīd (786-809 A.D.). Hārūn al-Rashīd, according to D.E. Smith:[63]

> ...was a great patron of learning. Under his influence several of the Greek classics in science, including part of Euclid's works, were translated into Arabic. Indeed, it is to the Arabic versions that medieval Europe was indebted for its first knowledge of Euclid's Elements. In his reign there was a second influx of Hindu learning into Baghdad, especially in the line of medicine and astrology.

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62. S.T. Arnold and A. Guillaume, The Legacy of Islam, p. 376.

63. D.E. Smith, History of Mathematics, Vol. I, p. 168.

This was continued during al-Māmūn (813-833 A.D.), but in his period some more fields like mathematics, astronomy, philosophy and others were added. The whole translation work of Greek sciences into Arabic was done by the Syrians, for they were familiar with the Greek language as well as Arabic and their scholars used to visit the Greek schools. There was another educational centre in the city of Ḥarrān in Mesopotamia which also played an important role in the translation. It is a great matter of fact that the term 'Arab' is used for all Muslims, either they were of Persia or of Turkistān, Khwārizm or of Spain, though the majority of the scholars were non-Arabs by birth. In 830 or 831 A.D. an observatory, 'Bait al-Ḥikmah' (house of wisdom) was created at Baghdad by the order of al-Māmūn for the purpose of astronomical observations which, later served as the translation centre also. In 851 A.D. a college was formed under the presidentship of Ḥonein ibn Isḥāq (192/808-260 A.H./873 A.D.). He and his son Isḥāq ibn Ḥonein translated a number of works into Arabic. Thābit ibn Qurra who is credited to be one of the earliest great scholars of the Arab world, published good works. The Arabs were so much fond of translating various sciences into Arabic that only in one century, after the establishment of Abbasid dynasty (upto

850 A.D.) the whole translation work including most of the medical works of Galen, Chief Philosophical works of Aristotle, the works of new platonic commentators, the Indian and Persian works, the works of Euclid, Archimedes ,Appolonius, Ptolemy and others was completed. By the year 950 A.D. they themselves acquainted with the ideas of algebraic notions and progress.

The following figure shows the transmission of ancient sciences through various languages into Arabic, its zenith and its translation from Arabic into Latin.

Aramaic Syriac → Translations ← Persian
Babylonian → Translations ← Sanscrit
Greek → Translations ← Koptic

The Muslim West
750 - 900 A.D.

The Muslim East
750 - 900 A.D.

The Muslim Golden Age
900 - 1100 A.D.
↓
Muslim Containment and Decline
↓
Translation into Latin
↓
Latin Medieval Renaissance
1100 - 1300 A.D.
↓
Rise of Universities
↓
The Great Renaissance
1300 - 1500 A.D.

Fig.1.

While the following figure gives the information of transmission of Egyptian, Babylonian, Indian and Chinese works through different civilizations.

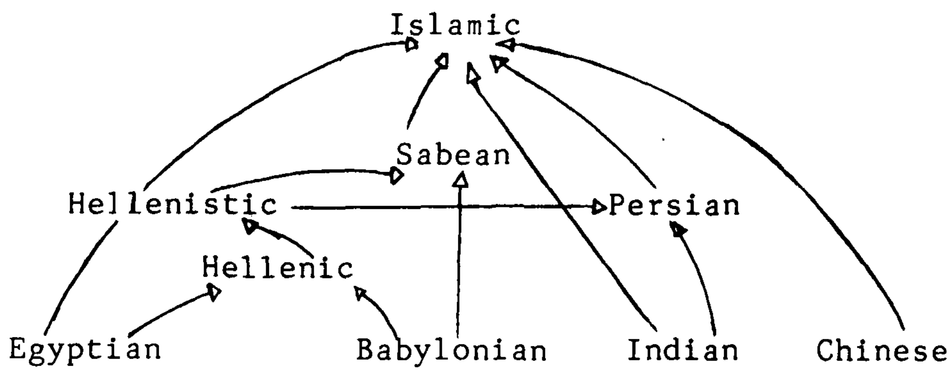

Fig. 2 : taken from Seyyed Hossein Nasr, 1976, <u>Islamic Science - An Illustrated Study</u>, World of Islam Festival Publishing Company Ltd., p.10.

Sedgwick and Tyler mention:[64]

> In the ninth century the school of Baghdad began to flourish just when the schools of Christendom were falling into decay in the West and into decrepitude in the East. The newly awakened Muslim intellect busied itself at first chiefly with mathematics and medical sciences; afterward Aristotle threw his spell upon it...(RASHDAL, UNIVERSITIES OF EUROPE, 1, 351).

Since the Indians and the Arabs had considerable commerce with each other therefore, the Arabs received and translated two of its great works of algebra like Siddhanta[65] which was introduced by an Indian traveller at Baghdad by the order of Caliph al-Manṣūr (754-775 A.D.). Among them was Muḥammad bin Ibrāhīm al-Fazārī (d.777 A.D.) who translated Siddhanta into Arabic and came to be known as first Muslim astronomer. It is notable that the numerals of Sindhind were called by the Europeans as Arabic numerals.

Muḥammad bin Mūsā al-Khwārizmī (780-850 A.D.), a native of Khurasan, visited Afghanistan and perhaps India, is a most leading personality among the early

64. W.T. Sedgwick and H.W. Tyler, op. cit., p.180.

65. An Indian astronomical treatise which Arabs called as Sindhind.

Muslim mathematicians. His algebra, Al-gebr wa'l-Muqābla (Restoration and Reduction) is based upon Indian and Greek mathematical work.[66] He also composed a treatise on arithmetic by the title Algoritmi De Numero Indorum, based on the principle of position, Hindu method of calculation and the origin of numerals as De Numero Indico. According to W.W. Rouse Ball " ...The subsequent Arab and early medieval works on algebra were founded on it [The algebra of al-Khwārizmī] and also through it the Arabic or Indian system of decimal numeration was introduced into West".[67] It was translated into Latin in the twelfth century by Gerard of Cremona and was being taught in Europe until sixteenth century A.D.

Thābit ibn Qurra, another great mathematician, especially a geometer following al-Khwārizmī, was born at Ḥarrān in 836 A.D. and died in 901 A.D. With the translations which he made of the works of Euclid, Appolonius, Archimedes and Ptolemy, he wrote several original works of which only a part of algebra is survived. His algebra consists of one chapter on cubic

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66. W.W. Rouse Ball, op. cit., p. 156.

67. Ibid.

equation solved with the help of geometry. Omar Khayyām (1045-1123 A.D)[68] was the first mathematician who solved the algebraic equation by intersecting cubics and obtained a root as abscissa of a point of intersection of a cubic and a circle as well as of a cubic section and another cubic section and divided cubics into two classes, the trinomial[69] and quadrinomial [70]. Praising him Smith says, "...he wrote on Euclid and on astronomy and contributed a noteworthy treatise on algebra".[71]

Abū 'Abdullāh Muḥammad bin Jābir bin Sinān al-Battānī (877 - 929 A.D.) wrote on astronomy with mathematics also. In his book Kitāb ma'rifat maṭāli' al-burūj fīmā baynā arba' al-falak (The book of the science on ascensins of the signs of the Zodiac in the spaces between the quadrants[72] of the celestial sphere) in which he deals the astrological problems with

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68. Mentioned by F. Cajori, op. cit., p. 107.

69. A Polynomial having three terms.

70. A polynomial having four terms.

71. D.E. Smith, op. cit., p. 286.

72. If a plane is divided by two mutually perpendicular straight lines into four equal parts then each part is called 'quadrant'.

mathematical solution. His De Scientia Stellarium gives trigonometrical term 'Sinus' (or Jayb) or half the chord in place of Ptolemy's whole chord and presented the table of 'Cotangents' for the first time in the history of mathematics. He also gives the trigonometrical solution of the astrological problems in another treatise. A commentary on Ptolemy's Tetrabilon is also ascribed to him.[73] He gave one side of a triangle in terms of the other sides and angle between them, for he knew the fundamental formula of spherical trigonometry. Abu'l- wafa al-Buzjānī (940 - 998 A.D.) was a geometrician who gave certain functions of Trigonometry and constructed tables of tangents and contangents, invented a method for computing tables of sines which gives the Sine of a half degree correct to nine decimal places and introduced the terms 'secent' and 'cosecent'. Among his mathematical works are :

(i) Fīmā Yahtaj ilayh al-Kuttāb wa'l-ummāt min 'ilm al-Ḥisāb on arithmetic;

(ii) al-Kāmil, about which Suter says with the reference of ibn al-Qifṭī that it is perhaps identical with Almgest;[74]

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73. C.A. Nallino, Encyclopaedia of Islam, (New Edition),Vol. I, p. 1104.

74. H. Suter, Encyclopaedia of Islam, Vol. I, p.158.

(iii) Al-Handasa which is not written by him and is thought to be a collection of his lectures. He also wrote commentaries on Euclid, Diophantus and al-Khwārizmī. He developed the study of trigonometry, worked on spherical trigonometry especially on right angled triangle. A portion of his writings is also devoted to geometrical construction. Al-Kūhī solved the problem by constructing the segments of a sphere. Al-Ṣaghānī and Abū'l- Rayḥān Muḥammad ibn Aḥmad al-Birūnī (937 - 1050 A.D.) worked on the trisection of angles. Abū'l-Jūd solved the problem by intersecting parabola with an equilateral hyperbola[75]. Ibn al-Haithām (987 - 1038 A.D.) of Egypt who, beside his commentaries on the text books of Euclid and the collection of problems[76] like the data of Euclid, computed the volumes of paraboloids formed by revolving a parabola about any diameter or any ordinate.[77] 'Abd al-Gehl who, perhaps died in 1100 A.D., wrote on conic sections and three small treatises on geometry. Abū Muḥammad al-Khojandī discovered that the sum of two cubes can never be a cube. Al-Karkhī

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75. A hyperbola whose transverse axis is equal to conjugate axis.

76. It was translated by L.A. Sidellot and published at Paris in 1836.

77. F. Cajori, op. cit., p. 109.

(d. 1029 A.D.) who flourished in eleventh century at Baghdād was one who contributed to the field of algebra by his glorious inventions in the field. He solved the guadratic equation for arithmetical as well as geometrical proofs and stood first to prove the theorems on the summation of the series as :

$$1^2 + 2^2 + 3^2 + \ldots\ldots + n^2 = (1+2+3+\ldots\ldots+n)\,\frac{2n+1}{3}$$

$$1^3 + 2^3 + 3^3 + \ldots\ldots + n^3 = (1+2+3\ldots\ldots+n)^2$$

The later mathematician, al-Kāshī, developing al-Karkhī's summation, gives the summation of fourth power.[78] Al-Māhānī is the first to state the cubic equation and proved it geometrically. Abū Ja'far Al-Khazīn solved the equation by conic sections. Naṣīr al-Dīn Ṭūsī was a persian who wrote treatises on algebra, geometry, arithmetic and translations of Enclid's Elements. He also worked on trigonometry independently. Ibn Yūnus (d. 1008 A.D.) solved the problems of spherical trigonometry.

In this way a number of Muslim scholars contributed to the field of mathematics and their school

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78. S.T. Arnold and A. Guillaume, op. cit., p.394.

flourished until fifteenth century. The Arabs taught the use of 'cipher' which they borrowed from Hindus and became the founders of the arithmetic of everyday life. Before the Arabs, algebra was not an exact science which was made by the Arab contribution and even developed to a higher level. The branch analytical geometry, plane and spherical trigonometry were founded by the Arabs. It is unfortunate that in the later period no great advancement in the field was done by them. W.W. Rouse Ball says that however the works of the Arabs stands second in quantity and quality as compared to that of the Greeks or modern Europeans but it is a fact that they catched the sciences very rapidly and did considerable work in the field in only 650 years.[79]

79. W.W. Rouse Ball, op. cit., p. 163.

C H A P T E R - II

SPANISH-MUSLIM MATHEMATICIANS OF EARLY MEDIEVAL PERIOD (10TH, 11TH AND 12TH CENTURIES A.D.)

The Muslim power emerged in Spain in the early decades of the eighth century A.D. and from that period onwards Spain became part of Umayyad Empire of Damascus. In 756 A.D. it started working as an independent state due to the establishment of Abbasid dynasty in the East. During the Umayyad rule, Spain became a great learning centre. Alongwith the development of knowledge in various fields, their development in architecture and agriculture was excellent which resulted a great economic satisfaction among its people. This satisfaction strengthened their efforts in creating some need works and developing them upto a higher degree.

It is worthy of note that the role of Islam and its civilization in Spain is most significant and Muslims contributed in all the aspects of life throughout their rule of about eight hundred years. They had been well-known in writing books on different topics, viz., history, biography, geography, agriculture, chemistry, botany, physics, medicine, and mathematics alongwith all their branches in arithmetic, geometry, algebra, trigonometry, surgery, pharmacy, as well as astronomy, music, poetry, prose and religious sciences as Quran, ḥadīth, fiqh and tafsīr.

In the 10th and 11th centuries A.D. Muslim learning in the country reached its zenith, schools and libraries were founded in the 10th century .A.D Sedgwick and Tyler quote Hume (pp. 102 and 109) as:[1]

> ...the schools and the libraries were famous throughout the world; Science and learning were cultivated and taught as never had been before. Jews and Muslims, in the friendly rivalry of letters, made their country illustrius for all times by the production of their study ... The schools of Cardova, Toledo, Seville and Saragossa attained a celebrity which subsequently attracted to them students from all parts of the world.

Thousands of schools and colleges were running and thousands of teachers were teaching during the Muslim rule in Spain. In the 12th century A.D. there were one thousand schools and colleges and more than ten thousand

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1. Sedgwick and Tyler, op. cit., p. 196

teachers in Cardova (Qarṭaba) only, and some of the scholars wrote hundred or more books and created great inventions of science.[2] The Universities of Granada (Gharnāṭa) Valensia (Blansiā), Cardova, Seville (Ishbīlia) and Toledo (Ṭalīṭala) were the great centres of learning. Spain is the scene through which almost all the works may they be of the Arabs, the Greeks or the Hindus were transmitted to Europe.

It has the fertilized soil which produced a number of great scholars in various fields. Al-Qālī (901-967 A.D.), a professor of Cardova University was one of the great scholars. Al-Zubaydī (928-89 A.D.) of Seville and al Suyūṭī were among those who were grammerians and linguists. The field of literature produced a great man, Ibn 'Abd-Rabbih (860-940 A.D.) at Cardova. Ibn Ḥazm (994-1064 A.D.) belonged to the group of great thinkers. Ibn Zaydūn (1003-1071 A.D.) contributed to the field of poetry. Ibn Khaldūn (1332-1406 A.D.) and ibn al-Khaṭīb (1313-1374 A.D.) were such great historians whose works are still the guides for the purpose of research in the field of historiography. Al-Idrīsī (1099-1166 A.D.) was one of the many great

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2. Rashīd Akhtar Nadvī, (nd), Musalmān Andalus Mayn, p. 746

geographers in the Muslim world. Ibn Zuhr (1091/94 - 1162 A.D.) and ibn Zahrāwī (d. 1013 A.D.) were two great surgeons and al-Ghāfiqī was famous as a botanist. Similarly, ibn Rushd (1126-1198 A.D.) ibn Maymūn (1135-1204 A.D.) and ibn Ṭufayl (d. 1185 A.D.)were great philosophers. Among the biographers, ibn al-Faraḍī (962-1013 A.D.) was a prominent scholar.

It is much difficult to say the exact date when the Eastern works were shifted to Spain, but inspite of all political rivalries between the umayyads of Spain and the Abbasids at Baghdad, they imported the Arab works, especially the works of Arab mathematicians as well as the Arabic translations of the Greek works such as of Euclid, Archimedes, Appolonius, Ptolemy and others. However, it is quite clear that al-Karmānī (d. 1066 A.D.) transmitted the mathematical ideas of Ikhwān al-Ṣafa[3] into Spain.[4] Al-Karmānī died in the year 1065 A.D.[5] or 1066 A.D.[6] at the age of 90. This shows that he transmitted these

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3. Brothern of purity, a group of scholars flourished in the East who wrote on different fields of learning including mathematics (see appendix 1).

4. Maulvī 'Abdul-Raḥmān Khān, Qurūn-e-wusṭā ke Musalmānon kī 'ilmi' Khidmat, Vol. I, p.163

5. ________, Dāirat al Ma'ārif al Sha'b, Vol. II, p. 197.

6. Maulvī A.R. Khān, op. cit., p. 188.

ideas probably in the second half of the 19th century A.D. Piya says, "...If the libraries of that period were not destroyed, a great number of books and a huge material on various subjects would have been found".[7]

Similarly Muslims of Spain played an important role in contributing to the field of mathematics whether it was arithmetic, algebra, trigonometry or geometry. In the words of Cajori, "...the Western Arabs took an advanced position in the matters of algebraic notations and were inferior to none of their predecessors or contemporaries, excpet the Hindus".[8] But at another place Cajori, regarding the algebraic symbolism says, "...Arabic albegra before him [Al-Qalaṣādī] contained much less symbolism than [Sic] Hindu algebra".[9] The Europeans have also realised their advancement of knowledge and ability in the field. They also benefited from their researches and works for a considerable period to a great extent.

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7. 'Abdul-Qawi Diya, Tarikh-e-Andalus Daur-e - Bani Umayyah Mayn, p. 972.

8. F. Cajori, op. cit., p. 111.

9. Ibid.

They used Ḥurūf al-Ghubār[10] which were different from the numerals used in the East. These numerals were invented near about 950 A.D. The word 'ghubār' is an Arabic word, means 'dust' which stands for written arithmetic with numerals, in contrast to mental arithmetic.[11] It is to be observed that mostly mathematicians of that period were astronomers who worked on mathematics and astronomy simultaneously. They, either created their own works or translated, corrected the earlier mathematical or astronomical works or both. Many of them wrote commentaries too.

Working in different fields of mathematics the Spanish-Muslim mathematicians framed the rules of division of decimals, used arithmetic in tracing the planatory motion and also fixing day and night, invented rules of equation. They also worked on commercial arithmetic and triangle as well as derived new theorems on trigonometry and other related topics.

However, due to insufficient information materials, it is impossible to present all the works and contribution of each and every Muslim mathematician

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10. Numerals introduced by Spanish-Muslim Mathematicians.

11. F. Cajori, op. cit., p. 110-111.

during the period of Muslim rule in Spain, but it is necessary to discuss all the scholars of mathematics whose accounts, through any source are available. The earliest mathematicians are reported to be living in the tenth century A.D. and in every period more than one scholar worked in this field till the downfall of the Muslim rule there in 1492 A.D. Amongst those mathematicians who, on their part stood as contributors were:

Al-Majrīṭī (d. 1007 A.D.)

Abū'l - Qāsim Maslamah ibn Aḥmad al-Faraḍī al-Majrīṭī is considered to be among the earliest and important Muslim scientists of Spain. Born at Majrīṭ (Madrid), most probably, in the second half of the 10th century A.D., he flourished at Cardova. All the writers are unanimous that he died in 1007 A.D.

Al-Majrīṭī was not only a mathematician but also an astronomer and chemist.[12] As an astronomer he worked on the astronomical tables of al-Khwārizmī and changed its meridian from Iran to Cardova. As far as his alchemy is concerned, though it seems to be spurious, three books

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12. As mentioned in Dā'irat al-Ma'ārif al-Sha'b, Vol. II, p. 195 and G. Sarton, Introduction to the History of Science, Vol. I, p. 668.

are ascribed to him: Rutbāt al-Ḥakīm (Sage's step); Ghāyat al-Ḥakīm (Aim of the wise) and Sirr al-Kīmiya.[13] About Ghāyat al-Ḥakīm Sarton says that its date was perhaps the middle of the eleventh century, while Juan vernet mentions that Rutbāt al Ḥakīm was composed after 1009 A.D.[14] Similarly, in Encyclopaedia of Islam J. vernet [15] writes that since Qāḍī ibn Ṣā'id does not mention him in his Ṭabqāt, therefore they may be wrongly attributed to al-Majrīṭī.

His intention towards the sake of knowledge may be recognized by the fact that in addition to getting the knowledge from a geometrician, 'Abd al-Ghāfir ibn Muḥammad, he joined a group of Hellenists in the period of 'Abdul Raḥmān III (912-961 A.D.). Apart from his alchemical and astronomical writings, he wrote a treatise on astrolabe and gave the knowledge of its technical construction and its use, a commentary on the planisphere of ptolemy in the name Taṣṭīh- e-Basīt al Kura' which is preserved in Latin version by Hermann of Dalmitia (1143 A.D.) and in Hebrew version also. He also went through al-Battānī's (877 - 929 A.D.)

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13. This third book is mentioned in Encyclopaedia of Islām (New edition), Vol. V, p. 1109.

14. G. Sarton, op. cit., p. 668 and Juan vernet, Dictionary of Scientific Biography, Vol.IX, p. 39.

15. J. Vernet, Encyclopaedia of Islam (E.I.), Vol.V, p. 1109.

theorems and emended some of them. Al-Majrīṭī is also credited to write some notes on the theorems of Menelaus in the name Shakl al-Khaṭṭa. Which is on geometry. He wrote Tamām'ilm al-a'dād, a text book on commercial arithmetic. In this book he adopted, according to ibn Khaldūn, arithmetical, geometrical and algebraic techniques altogether in connection with dealing sales, valuation and taxation.[16] Since the Muslims were dominating on the business almost all around the world therefore, they needed some new rules useful in their business. Al-Majrīṭī, keeping their need in mind, introduced some new methods and rules. These methods and rules were compiled in one book. With this effort al-Majrīṭī made commercial arithmetic , a complete science. This text book of al-Majrīṭī is called al Mu'āmlāt or Tamām'ilm al-a'dād. He was also acquainted with mensuration and amicable numbers[17] and expressed that these numbers have etoric power. Iamblichus (flourished about 320 A.D.), a renown philosopher of Neoplatonic school mentioned that the amicable numbers were discovered by pythagoras.[18]

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16. Juan vernet, op. cit., and J. vernet, op.cit.

17. A pair of numbers is amicable with the condition that the sum of the factors of one equals to the other, and conversely.

18. Howard Eve., An Introduction to History of Mathematics, p. 55.

In response of his mathematical writings he was known as "al-Ḥāsib" (the mathematician.[19] He or his pupil, al-Karmānī (d. 1066 A.D.) was also credited to introduce the Rasā'il of Ikhwān al-Ṣafā into Spain, but it seems to be clear from various writings that he is the only man to take these Rasā'il to Saragossa and other northern parts of Spain.

Al-Majrīṭī exercised great influence on mathematical and astronomical learnings in Spain through his own writings as well as his pupils, as they settled in various parts of Spain. Abū'l Qāsim Aṣbagh ibn al-Samḥ (979-1035 A.D.) migrated from Cardova to Granada; Abū'l Qāsim Aḥmad ibn al-Ṣaffār (d. 426/1035) flourished at Cardova but later on went to Denia; ibn al-Khayyāṭ (d. 447/1055 A.D.) who worked as an astrologer and Abū Muslim ibn Khaldūn lived in Seville, were all his pupils who, as great scholars of that time contributed to mathematical learning at their respective flourishing places. There is no mathematician of Muslim Spain parallel to al-Majrīṭī who produced such scholars in the field of mathematics in quality and quantity.[20]

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19. P.K. Hitti, History of the Arabs, p.570.

20. Ibn Sāid Andalusī, Kitāb-e-Ṭabqāt al-Umam, p. 69.

Ibn al-Samḥ (979-1035 A.D.)

Abū'l-Qāsim Aṣbagh ibn Muḥammad ibn al-Samḥ was a famous mathematician and astronomer. He was born in 979 A.D. in Qarṭaba (Cardova).[21] At his native place he got his education under the famous scholar of that period, Abū'l-Qāsim Maslamah al-Majrīṭī (d. 1007 A.D.). Under his able guidance he got the knowledge of mathematics and astronomy. But, later on in the beginning of 5th/11th century he had to migrate to Granada which was in those days under the rule of Hābūs bin Maksān under whose protection ibn al-Samḥ could carryon his mission of writing. There, in Granada ibn al-Samḥ died in 426 A.H./1035 A.D.

Ibn al-Samḥ was considered to be a great scholar of his time who became familiar due to his own writings. His work on astronomy is both theoretical as well as practical. Apart from his mathematical writings he compiled the astronomical tables and wrote on astronomical instruments. In his tables he adopted the method used in Sidhanta (Ar. Sind hind) and explained them theoretically.[22]

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21. D. Pingree quotes Ibn al-Abbār in Encyclopaedia of Islam (New Edition), Vol. III, p. 928.

22. George Sarton, op. cit., Vol. I, p. 715.

Ibn al-Samḥ wrote in the field of mathematics on arithmetic and geometry as well as on mental calculus. As a geometrician he worked on straight line, curves and arcs.[23] His mathematical works are :[24]

1. Kitāb al-Madkhal ilā al-Handasa fī Tafsīr al-Uqlidūs;
2. Thimār al-a'dād, which is also known as al-Mu'amlāt, but different from the Mu'amlāt of al-Majrīṭī.
3. Kitāb Ṭabī'at al-a'dād.
4. Kitāb al-Kabīr fī al-Handasa.

Following are the works of ibn al-Samḥ on astrolabe.

1. A book which is based on two maqalas for the purpose of making astrolabe.
2. Another book of ibn al-Samḥ explains the use of astrolabe. This book is so much bulky that it contains 130 chapters.
3. On the model of Zīj of Sindhind he composed his own work. This book is divided into two parts:

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23. Ibn Ṣā'id Andalusī, op. cit., pp. 69-70.

24. Cf. Hājji Khalīfa, Kashf al-Dhunun quoted by D. Pingree, op. cit.

the first contains all of his tables while in the second part he explains these tables. Al-Zarqālī has mentioned in his Kitāb al-a'māl bi al-Ṣafīḥa (in chapters 63, 64, 65) the method of ibn al-Samḥ of equalizing the astrological places, projection of rays and rising of the stars.[25]

Ibn al-Ṣaffār (d. 1035 A.D.) :

Abū'l- Qāsim Ahmad bin 'Abdullāh bin'Umar al-Ghāfiqī al-Andalusī was among the pupils of Maslama al-Majrīṭī (d. 1007 A.D.). He received the knowledge of mathematics and astronomy from this famous and great scholar, and later worked on these fields before and after his migration to Denia from Cardova. Due to the lack of more information, a very few is known about his life and intellectual activities. According to sources, he was born in Cardova but its date lies in dark. He flourished there in Cardova but due to civil war there, his intellectual life was disturbed. This civil war became the reason for his migration to Denia where he died in the year 426 A.H./1035 A.D.

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25. Cf. Al-Zarqālī, Kitāb al-A'māl bi'l-Ṣafīḥa in E.I. (New Edition) by D. Pingree, op.cit.

Informations of only a treatise on the use of astrolabe by him and of the compilation of astronomical tables on the method of Sindhind are available.[26] His brother, Muḥammad is also considered to be an astrolabe-maker, one of which is still surviving. No more details of the works of ibn al-Ṣaffār are available. His tables are survived in an Arabic manuscript while the treatise on astrolabe was later edited by J.M. Millas Valincrosa. This treatise has its two latin versions, by Johennes Hispalensis, and by Plato of Tivoli.[27]

Ibn Abī al-Rijāl or al-Rijāl (d.432 A.H./1040 A.D.)

Abū'l-Hasan 'Alī ibn Abī al-Rijāl al-Ṣaibānī al-Kātib al-Maghribī was, according to Cyril Glasse, "...a mathematician, astrologer and astronomer". But Sartom mentions him as an astrologer.[28] He was born most probably in Cardova but its date is not determined. He, however, flourished in Tunis (1016-1040 A.D.,) and died there in or after 432 A.H./1040 A.D.[29]

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26. Cf. Ṣā'id al-Andalusī in Encyclopaedia of Islam (New Edition) by B.R. Goldstein, Vol. III, p. 924.

27. B.R. Goldstein, op. cit.

28. Cyril Glasse, Concise Encyclopaedia of Islam, p. 336 and G. Sarton, op. cit., Vol. I, p. 716.

29. Cyril Glasse, op. cit., and G. Sarton op. cit.

Ibn Abī al-Rijāl's work on mathematics is not survived but one of his astrological work al Bāri'fī Aḥkām al Nujum, is a distinguished book on horoscopes from the constellations. His other books are mentioned in Fihris al-Makhṭūṭāt al-Musawwarah entitled Sharaḥ Manzūmah ibn Abī al-Rijāl fi Ahkām al-Falāk, preserved in Aleppo and Sharaḥ Qaṣīdah ibn Abī al-Rijāl fī Aḥkām al-Nujūm in Damascus. Al-Bari' is his main work which was translated by Judah ben Moses from Arabic into castillian and from Castillian into Latin by Aegidius de Tebaldis and Petrus de Regio.

Al-Karmānī (d. 1066 A.D.)

Abū'l-Ḥakam Amr (or 'umar) ibn 'Abd al-Raḥman ibn Aḥmad ibn 'Alī al-Karmānī was also among the early mathematicians of Muslim Spain. He was born in Cardova, lived for some period in the East and died in 1066 A.D. in Saragossa after passing 90 years of his life.[30]

Al-Karmānī, having the engagements of being a surgeon, devoted a considerable time to mathematics. Unfortunately we are unable to have any of his mathematical writings but it is being said that he or his

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30. G. Sarton, op. cit., Vol. I, p. 715 and M. Mun'im Khaffāja, Qissat al-Adab fī'l Andalus, p. 161.

teacher, Maslamah al-Majrīṭī introduced the Rasā'il Ikhwān al Ṣafā (writings of brethren of purity) into Spain.[31]

Al-Jayyānī (b. 989/990 d. after 1079 A.D.)

Abū 'Abdullāh Muḥammad ibn Mu'adh al-Jayyānī was a scholar who worked in the fields of Arabic philology, inheritance laws (farā'iḍ) and mathematics as well as on astronomy and Qur'āniyāt (the knowledge of Quran). He was known as 'Qāḍī' (Judge) and 'faqīh' (Jurist).[32] Since he flourished in Jaen,[33] he was called al-Jayyānī. It is most probable that he was born in Cardova in 989/990 A.D. and died after 1079 A.D. It is due to the fact that he wrote his treatise On the total Solar Eclipse in 1079 A.D.

Many astronomical and mathematical works are reported to have been written by al-Jayyānī. Though the Arabic texts of the works On the total Solar Eclipse and On the Dawn are not available now, but the translations of both of these were made into Hebrew by Samuel ben

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31. P.K. Hitti, op. cit., p. 571.

32. Cf. Ibn Bashkuwal, Maqālah fī Sharḥ al-Nisba, by Y.D. Samplonius and H. Hermelink in D.S.B, Vol. VII, p. 82.

33. Jaen is the capital of a province of the same name in Spain.

Jehuda and the later into Latin by Gerard of Cremona entitled Liber de Crepusculis. He compiled and sumplified the work Tabulae Jahen of which latin translation was made by Gerard of Cremona under the title Liber tabularum Jahen cum regulis Suis. Tabula residum ascensionum ad revolutiones annorum Solarium Secundum Muhad Arcadius and Maṭrah Shu'āat al-Kawākib are other astronomical works ascribed to al-Jayyānī. The mathematical works of al-Jayyānī are Kitāb majhūlāt qisiyy al-Kura, (determination of the magnitudes of the arcs on the surface of a sphere) on spherical trigonomery and a treatise entitled, On Ratio. In response to his mathematical writings ibn Rushd mentioned al-Jayyānī as a great mathematician.

In his Tabulae Jahen al-Jayyānī shows that the difference between real astronomical and average date according to the Islamic lunar calander may be as much as two days.[34] The work was based on the tables of al-Khwārizmī and its tables were converted to the longitude of Jaen at the midnight of 16th July 622 A.D. To determine such things as the direction of the meridian, the visibility of new moon, the time of the day

34. Cf. Tabulae Jahen of al-Jayyānī in D.S.B., quoted by Y.D. Samplonius N. and Hermelink, Vol. VII , p. 82.

as well as the time and direction of prayer, the calander, the prediction of eclipses and the setting up of horoscope, al-Jayyānī gave clear instructions in this book. After going through the theories of al-Khwārizmī and Ptolemy he rejected them on the division of houses (manāzil). He also rejected the theory of ray emission of Abū Ma'shar.

In *Liber de Crepusculis*, al-Jayyānī mentioned about morning and evening twilight and gave the angle of depression of The Sun at the beginning of the morning twilight and at the end of evening twilight as 18^{o}. As the twilight happens because of the moisture in atmosphere, al-Jayyānī tried to find out the height of this atmospheric moisture with the help of the angle of depression. He supposed the angle as the fourth magnitude alongwith the body, surface and the line.[35] In the treatise *On Ratio*, he defines five magnitudes which are used in geometry. They are number, line, surface, angle and solid. In this connection the writer says "...The un-Greek view of considering number an element of geometry is needed here because al-Jayyānī bases his definition of ratio on magnitudes".[36]

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35. Cf. Ibn Rushd, *Tafsīr* (II p. 665) by Samplonius and Hermelink, *op. cit*.

36. Y.D. Samplonius and H. Hermelink, *op. cit.*, p. 83.

Al-Jayyānī sited Euclid and commented those Arab mathematicians who objected on his book V definition 5. He says in the preface of his On Ratio that its purpose "...to explain what may not be clear in the fifth book of Euclid's writing to such as are not satisfied with it".[37] He enjoys the status parallel to Issac Barrow in connection with the understanding the Euclid's book V.

Al-Jayyānī was among the scholars who chose the way either 'to find a relation between the views of the Arab mathematicians and the unsatisfying theory of Euclid's doctrine of proportion' or 'to obtain equivalent results more in accord with their views'.[38] He, in this regard, criticized the Arabs not to understand his doctrine and to deduce little or nothing from his book. He established a common base regarding the conception of ratio and proportionality and then derived many characteristics of proportional magnitudes. By converting the Euclid's multiples into parts he made connection among them. With this process the magnitudes become truely proportional that satisfies Euclid's criterion. Al-Jayyānī based a fourth proportional and the unlimited

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37. Quoted from the preface of al-Jayyānī's, On Ratio in D.S.B., op. cit.

38. Ibid.

divisibility of magnitudes to prove its converse by an indirect proof. He discusses unequal ratios in the third part of this book.

Yūsuf al-Mu'tamin

Yūsuf al-Mu'tamin[39] was such a scholar of Muslim Spain who, inspite of having the burden of running the government upon his shoulders, devoted a considerable time to learning, especially, mathematics. He, as well as his father, Aḥmad al-Muqtadir billāh who ruled at Saragossa (Sarghosa) from 1046 to 1081 A.D., were great patrons of learned personalities of their state. Though his exact dates of birth and death are not confirmed but certainly he ruled at Saragossa from 1031 to 1085 A.D. after the death of his father.[40]

Being a mathematician he is known to write a treatise on the subject in the name Istikmāl which meant 'bringing to perfection'. Unfortunately, this treatise like some other works of different writers has not survived today. Joseph (Yūsuf) ben Judah ibn Aqnin, showing the importance of this treatise, writes that

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39. He flourished in the second half of the eleventh century.

40. G. Sarton, op. cit., Vol. I, p. 759.

this treatise should be studied together with Euclid's the Almagest and other middle books of importance.[41] Seeing the importance of this work it may be supposed that there might be some other works of Yūsuf al-Mu'tamin which were destroyed by the cruel hands of time after the downfall of Muslim rule in Spain.

Al-Zarqālī (1029 - 1087 A.D.)

Abū Isḥāq Ibrāhīm ibn Yaḥyā al-Naqqāsh al-Zarqālī was basically an astronomer, but he also worked as a mathematician especially in the field of trigonometry. He was born in 1029 A.D. in Cardova, flourished there, continued his literary pursuits and made observations, according to Sarton in 1061 and 1080 A.D. He died in 1087 A.D.

Al-Zarqālī is considered as a renowned astronomer of Muslim Spain who worked in this field more accurately than earlier ones. He is credited with having invented an improved version of astrolabe 'Ṣafīḥah'[42] and produced a work, containing the description of 'Ṣafīḥah' which was later translated into Latin, Hebrew

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41. Maulvī 'Abdul-Raḥmān Khān, op. cit., Vol. I, p. 225.

42. P.K. Hitti, op. cit., p. 572 cf. Khwārizmī, Mafātīḥ, p. 233-234 (It is a flatsheet or slab or anything, wood, stone, metal etc.

and other languages; to prove the motion of the solar apogee with reference to stars; to make observations of the obliquity of ecliptic and to edit the Toledan Tables [43].

As far as the trigonometry of al-Zarqālī is concerned, he wrote a separate work on its introduction. The work is titled as Canones sive regulae tabularum astronomiae in which he explains the construction of the trigonometrical tables.[44]

Jābir ibn Aflaḥ (d. 1140 A.D.)

Abū Muḥammad (or Abū Maḥmūd) Jābir ibn Aflaḥ (in Latin: Geber) was a great mathematician, astronomer and chemist. His exact date of birth is yet to be traced but according to U.R. Kaḥḥālah, he was born in Seville and died in Cardova some time in the middle of the 12th century.[45] It is an established fact that he flourished in Seville for a long period but there is no unanimity among different reports from different sources regarding

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43. These are the planatory tables based on the observations made by him and other astronomers at Toledo.

44. G. Sarton, op. cit., Vol. I, p. 759.

45. U.R. Kaḥḥālah, 1960 A.D. Mu'jam al-Muallifīn, Vol.,III.P.105.

his date of death. Sarton says that he died "...probably about the middle of thirteenth century", while H. Suter[46] and R.P. Lorch[47] mention that he flourished in the first half of 12th century A.D. The later statement seems to be correct because he flourished during the period of Ibn Maymūn or Maimonides (1135 - 1204 A.D.), a Jewish philosopher who flourished in Egypt, holding the post of physician of the ruler Ṣaladīn (1174-93 A.D.).

Jābir ibn Aflaḥ wrote an important work on Ptolemy's <u>Qānūn al-magisṭī</u> in 9 books. This work of Jābir has two different titles: <u>Iṣlāḥ al-Majisṭī</u> (corection of <u>Almagest</u>); and <u>Kitāb al-Hai'ā</u> (The book of astronomy). <u>Iṣlāḥ</u> is name of that copy which is an Arabic manuscript preserved in Berlin, while <u>Hai'ā</u> is the title of another copy which is mentioned in the <u>Bibliotheca Escurial Spain</u>.

Though the trigonometrical works of Jābir are of the same rank or inferior to Abū'l Wafa (940 - 997/98 A.D.) but he is more popular than Abū'l Wafa due to the translations of his <u>Iṣlāḥ</u> into Latin as well as Hebrew. The original books as well as their translations were available and used till the 17th century A.D. Gerard of Cremona (into Latin) and Moses ibn Tibbon (into Hebrew in

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46. H. Suter, <u>Encyclopaedia of Islam</u>, Vol.II,p.157.
47. R.P. Lorch, <u>D.S.B.</u>, Vol. VII, p. 38.

1274 A.D.) are the earlier translators of the work of Jābir ibn Aflaḥ. Moses' nephew, Jacob ben Mahir again translated it in the second half of the 13th century which was revised by Samuel ben Judah 1335 A.D. Inspite of the fact that he was quoted by al-Bitrūjī (12th century A.D.) and ibn Rushd in his Compendium of Almagest as well as his Iṣlāḥ was summerized by Quṭub al-Dīn al-Shīrāzī (13th century A.D.),the translation done by Gerard made him more familar in the West and that was commonly utilized by the orientalists in Europe.

In his Iṣlāḥ al-Majisṭī, Jābir criticised the views of ptolemy regarding planets, that the lower planets mercury and venus have no visible parallexes and venus may lie on the line between the sun and the earth. He placed venus and mercury above the sun, not in a line. Whatever the errors Jābir found in ptolemy's Almagest, he listed them all in the prologue of his Iṣlāḥ.

Jābir discussed the introduction of trigonometry and gave for the first time the fifth main formula for the right angled spherical triangle (i.e. CosA = Cosa Sin B). He derived the formulae on the basis of the 'rules of the four magnitudes'. He was well acquainted with the theorem that 'the sines of the angles of a spherical triangle are proportional to the Sines of the opposite sides'.[48] The problems of plane trigonometry were solved by him with the help of whole chord, not Sine and Cosine.

Jābir replaced Menelaus' theorem by the theorems on right spherical triangles, therefore he based his spherical trigonometry on the rule of four quantities instead of six quantities. Unlike ptolemy who presented his theorems in the form of numerical examples, Jābir was a theorist and looks to be same as Abū'l-Wafa but according to Lorch his spherical trigonometry is less elaborate than that of Abū'l-Wafa.[49] Jābir's first book contained four theorems (12-15) on the basis of a theorem which gives the idea that the sides of a spherical triangle may be found out by the Sines if they are greater or less than a quadrant. His theorems may be written separately in modern notations as:[50]

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48. Cf. The Translation of Jābir's Work by Gerard of Cremona by W.W. Rouse Ball, <u>op. cit</u>., p. 165.

49. R.P. Lorch, <u>op. cit</u>.

50. Ibid.

<u>Theorem 12</u> :

If all the lines in a figure are arcs of great circles, then

$$\frac{\text{Sin } XQ}{\text{Sin } QP} = \frac{\text{Sin } XB}{\text{Sin } BM} = \frac{\text{Sin } XZ}{\text{Sin } ZY}$$

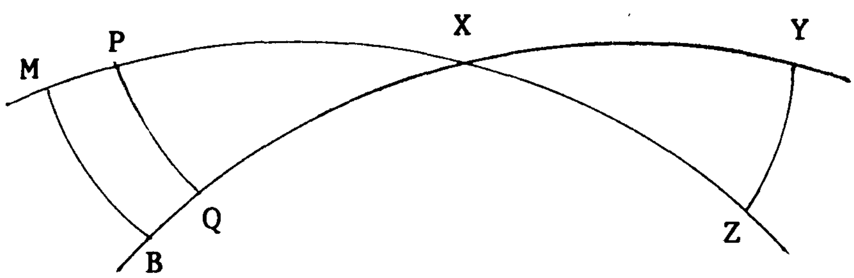

FIG. 3

Theorem 13 : If XYZ is a spherical triangle, then

$$\frac{\text{Sin } YZ}{\text{Sin } \angle X} = \frac{\text{Sin } ZX}{\text{Sin } \angle Y} = \frac{\text{Sin } XY}{\text{Sin } \angle Z}$$ ($\angle$ denotes angle)

Theorem 14 : PQR is a spherical triangle where $\angle Q = 90^\circ$, then

$$\frac{\text{Sin } \angle P}{\text{Sin } \angle Q} = \frac{\text{Cos } \angle R}{\text{Cos } PQ}$$

Theorem 15 : MNO is a spherical triangle and $\angle N = 90^\circ$, then

$$\frac{\text{Cos } MO}{\text{Cos } NO} = \frac{\text{Cos } MN}{\text{Sin (quadrant)}}$$

As far as his method in solving the problems is concerned, it was his own but not similar as Abū'l Wafa who was also a great scholar of trigonometry, flourished before Jābir in Buzjan and then Iraq in the second half of 10th century. He might have got some source material from Thābit ibn Qurrā's treatise on Menelaus' theorem or directly from Menelaus' *spherics*. A Torquetum- like instruments (Turquet) is said to have been described by Jābir which, according to him, is the replacement of all the instruments related to *Almagest*.

Jābir ibn Aflaḥ is a great criticizer of the work of ptolemy. He, therefore, exercised great influence by his trigonometry upon the whole West. He also gained popularity due to the writings of a number of Western scholars who, in their works utilized his Iṣlāḥ al-Majisṭī. Richard of Wallingford used it in his Albion and De Sectore while Simon Bredom discussed it for a long in his commentary on Almagest. His theorems are made more general in a part of another commentary on Iṣlāḥ. Regiomontanus' work on trigonometry,De Triangulis (written in 1460's and Published in 1533 A.D.) is most influenced by Iṣlāḥ which, for the first time was systematized trigonometry in Latin West. He was being quoted by the Western scholars even in the 16th and 17th centuries A.D. that may be regarded as a great proof of the importance of his writings. Henry Seville and Pedro Nunez are among those who studied and quoted him in their works. Corpernicus spherical trigonometry is the same work as Iṣlāḥ, who called Jabir as 'egragious calumniator of Ptolemy'.[51]

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51. Ibid., p. 39.

Ibn Rushd (1126 - 1198 A.D.)

Abū'l-Walīd Muḥammad bin Aḥmad bin Muḥammad ibn Rushd (Latin :Averroes) is a well known scholar of Spain who, as some other great scholars of Muslim world, worked in different fields of Qur'ānic Sciences and also on other subjects as philosophy, physics, biology, medicine, astronomy and mathematics. According to his biographical accounts, he was born at Cardova in 1126 A.D. and died at Marrakash in 1198 A.D. The educational background of ibn Rushd is reported to be very sound. His grandfather was a famous qāḍī and imām at Cardova mosque. His father also holded the post of qāḍī there. He received his education of different fields from various scholars of Spain. His teacher of juridical education was al-Ḥāfiẓ Abū Muḥammad ibn Rizq while he learned the science of the traditions of the Prophet (p.b.u.h.) from ibn Bashkuwāl. In the fields of medicine and philosophy he received knowledge from Abū Ja'far Hārūn al-Tujībī. He studied under some other scholars also. He made a number of astronomical observations. He wrote on Metaphysics and dealt astronomy physically as well as mathematically.[52]

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52. D.E. Smith, op. cit., p. 208.

A large number of works on different subjects are reported to be written by ibn Rushd. Unfotunately most of his original works are lost and only a few are survived in Arabic manuscript. However his fiftytwo works are survived either in original or translation form in Latin or Hebrew.[53]

In the field of mathematics, ibn Rushd wrote on trigonometry but no work in this field is survived.

Abū'l-Ṣalt (1067 - 1134 A.D.)

Abū'l-Ṣalt Umayyah bin 'Abd al-'Azīz ibn Abī'l-Ṣalt al-Andalusī was one of the great scholars who worked in the fields of medicine, mathematics and astronomy altogether. According to the reports, he was born in 460 A.H./1067 A.D. in Denia, Spain.[54] The first 29 years of Abū'l-Ṣalt were passed in Spain as he lived in Denia and then Seville. In his native country he received a vast knowledge from Qāḍī al-Waqqāshī. Then, for the sake of further knowledge, he migrated to Egypt in the year 489 A.H./1096 A.D. In Egypt he stayed at

53. Urdu Encyclopaedia of Islam, (Edited under the auspices of The University of Panjab, Lahore, Vol. I, p.525.

54. G. Sarton, op. cit., Vol.II, Part I, p. 230 and S.M. Stern, Encyclopaedia of Islam, Vol. I, p. 149.

Cairo and Alexandria where he remained busy in his studies. Unfortunately he had to leave Egypt in 505 A.H./1192 A.D. due to the failure in lifting a sunked ship at Alexandria and arrived at Mahdiya, Tunis. He died in Madhiya in 529 A.H./ 1134 A.D.

So many works of Abū'l-Ṣalt dealing with logic, music, astronomy, physics, cosmography, mathematics and medicine are mentioned in various books of Arabic.[55] In the field of mathematical science he wrote on geometry. A work al-Rasā'il al-Miṣriya is attributed to him which contains the information about the people of Egypt. A very few part of the work on mathematics by Abū'l Ṣalt is available. One of his writings, Taqwīm al-dhihn (Rectification of the understanding) deals with logic; Risālah fī'l-A'māl bī'l-Asṭurlāb provides the mechanism and use of Astrolabe; al-Adwiya al-Mufrada is written by him on simple drug; Risālah fī'l-mūsīqī on music and a summary of astronomy are also attributed to him. He also wrote a treatise in the form of answer to different problems of mathematics with its short summary.[56] This treatise of Abū'l Ṣalt also deals with the problems

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55. H.G. Former, Legacy of Islam (Urdu translation) Edited by Sir Thomus Arnold and Alfred Goyume, translated by 'Abdul Majeed Sālik, pp. 509-515.

56. S.M. Stern, op. cit.

of cosmography and physics. The title of this treatise is not available now.

In this way it may be concluded that Abū'l-Ṣalt got a master-mind who was expert of different subjects and wrote on those subjects in a very convincing manner.

Ibn al-Yāsmīnī (d. 1204 A.D.)

Abū Muḥammad 'Abdullah ibn Muḥammad ibn Ḥajjāj ibn al-Yāsmīnī al-Adrīnī al-Ishbīlī was a mathematician, different from other mathematicians of Muslim Spain. He was not an Arab but of Berber origin. Due to unavailability of detailed data it is impossible to mention his date of birth. However, he or his family migrated from Fās (North Africa) to Spain and partly flourished in Morocco and Seville.[57] His life was ended due to strengeling in Morocco in the year 1204 A.D.

Ibn al-Yāsmīnī is known as a mathematician who wrote his treatise on algebra al-arjūza al-Yāsmīnīya in the form of poem, of which according to Sarton "...there are many manuscripts".[58] No other work of ibn al-Yāsmīnī on mathematics is reported to be written. It is also not

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57. G. Sarton, op. cit., Vol. II, Part I, p. 400.

58. Ibid.

known what type of work on algebra was done by him in his al-arjūza.

Muḥyī al-Dīn al-Maghribī

Muḥyī al-Dīn al-Maghribī is one of the greatest Muslim mathematicians and astronomers who, in the company of Naṣīr al-Dīn Ṭūsī (1201 - 1274 A.D.)[59] and his fellow-men made a number of astronomical observations at Marāgha. A very few information about his life is available. He was born elsewhere in Spain, but still it is difficult to say about his exact dates and places of birth and death. Apart from Spain he flourished for sometimes in Syria and Marāgha (1260 - 1265)[60] where he met Naṣīr al-Dīn Ṭūsī and worked with him.

Apart from the astronomical observations and writings on it, he worked on mathematics especially trigonometry, astrology, chronology and revised and edited a number of Greek works. He and Naṣīr al-Dīn al-Ṭūsī were much praised by Maulvī A.R. Khān. S. Tekeli gives the following list as the works of Muḥyī al-Dīn al-Maghribī.[61]

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59. T. Arnold and A. Guillaume, The Legacy of Islam, p. 393.

60. S. Tekeli, D.S.B., Vol. IX, p. 555.

61. S. Tekeli, op. cit., Vol. IX, p. 555.

1. Kitāb Shakl al-qaṭṭa', a book on the theorems of Menelaus. Al-Maghribī, in this book discussed, described and corrected the Menelaus' theorems.

2. Mayanfari'u 'an Shakl al-qaṭṭa' (Consequences deducted from Shakl al-qaṭṭa'), that whatever the consequences were there in the views of al-Maghribī, they all were mentioned in this book.

3. Risālah fī Kayfiyāt istikhrāj al-Juyūb al-Wāqi'a fī'l-dā'irah (Treatise on the calcula tion of Sines).

4. Khulāṣa al-Majisṭī (Essence of Almagest). New determination of the obliquity of the ecliptic made at Maragha, is discussed in this book by al-Maghribī, in 1264, 23; 30^{o} while the real value in 1250, according to Tekeli was 23; 32, 19^{o}.

5. Maqālah fī istikhraj ta'dīl al-Nahār wa sa'at al-mashriq wa'l-dā'ir min al-falak (Treatise on finding the meridian, ortive amplitude and revolution of the sphere).

6. <u>Muqaddimat tat'allaqū bi ḥarakāt al-Kawākib</u> (premises on the motions of the stars).

7. <u>Tastīḥ al-asṭurlāb</u> (The flattening of the astrolabe).

The first three above mentioned books are written by al-Maghribī on trigonometry and the remaining are on astronomy. He revised and edited the following four Greek books:

1. Euclid's <u>Elements</u>;
2. Appolonius' <u>Conics</u>;
3. Theodsius' <u>spherics</u>;
4. Menelaus' <u>spherics</u>

Six books on astrology and one on chronology are also attributed to al-Maghribī.[62]

There are some other contributions of Muḥyī al-Dīn al-Maghribī in various fields of trigonometry. As far as right angled spherical triangles are concerned, he gave two proofs related to Sine, gave the value of Sin 1^{o}, the ratio of the circumference of a circle to its diameter, determined two mean proportionals between two lines, and on other triangles he generalized one theorem.

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62. Ibid.

In finding out the value of Sine 1^{o} he used the theorem that 'the differences of Sines of arc having the same origin and equal differences become smaller as the arc becomes larger', explained by Abū'l-Wafa (328/940 - 388/998)[63]. Earlier *Ptolemy calculated* approximately equivalent value of Chord 1^{o}. Then following him and with the help of cubic equation, al-Kāshī (d. 1429/1430 A.D.) found its exact value. He and Abu'l-wafa tried to find the value of Sine of one-third of an arc. With the help of the figure given below al-Maghribī calculated Sin 1^{o}.

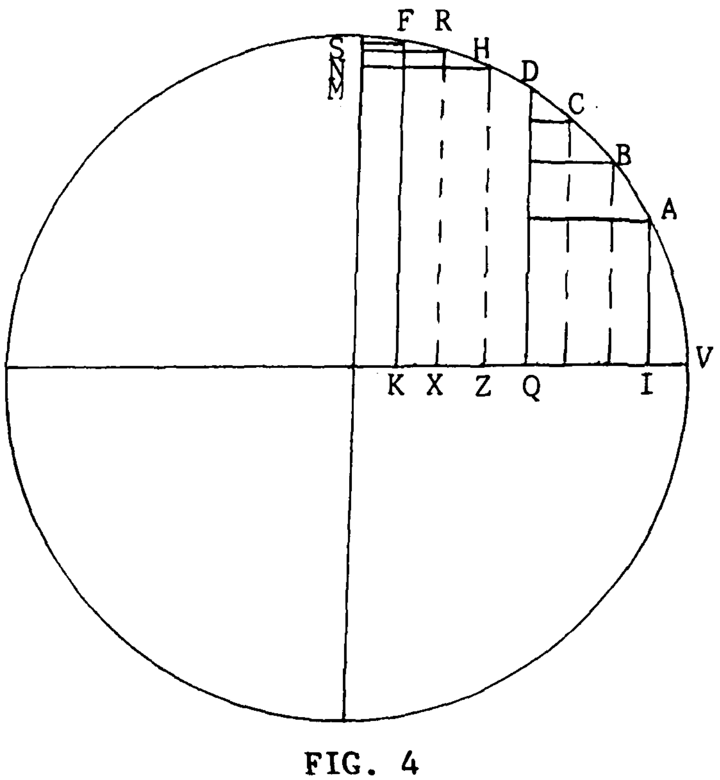

FIG. 4

63. Ibid.

The arc VF = 1; 7, 30^{o} and Sin VF = FK = 1;10,40,12,34^{P}. Similarly AV = 0; 45^{o} and Sin AV = AI = 0; 44,8,221,8,38^{P}.

When the arc AF is divided into six equal parts and each part = 0; 3, 45^{o}; then arc DV + arc DH = 1^{o} and Sin HV (=1^{o}) = HZ

The perpendiculars AT, BY and CK on DQ divide DT into three equal parts:

TY > YK > DK; TD/3 > HL;

DQ + TD/3 (=1; 2, 49, 43, 36, 9^{P}) > HZ (=Sin 1^{o})

Similarly, FM is divided into three equal parts:

MN > NS > SF; DQ + FM/3 (=1; 2, 49, 42, 50, 40,40^{P}).

The ratio of the circumsferences of a circle to its dimater (π) was calculated by al-Maghribī by using the method of interpolation based on the ratio of arcs greater than the ratio of Sines. He, following the above method of interpolation found Sin 1^{o} = 1; 2, 49, 42, 17, 15, 12^{P} and then found the difference between two values of Sines is 0; 0,0,0, 56^{P}, which according to S. Tekeli, "... is correct upto four places".[64]

64. Ibid., p. 556.

Arc ABC = $\frac{AC + RF}{2}$ = 1; 34, 14, 16, 47, 19, 30^P

The circumference = 240. Arc AB = 6; 16, 59, 47, 18^P, when the diameter is taken as 2^P. If the diameter being 1^P, the circumference = 3, 8, 29, 53, 34, 39^P < 3R +1/7, Sin 1/7 = 0; 8, 34, 17, 8, 34, 17.^{P}

Al-Maghribī found the length of regular polygons of ninety six sides which are inscribed and circumscribed. Later on with the help of above value and the value described by Archimedes he got :

3R + 1/7 < the circumference < 3R + 10/71

and described the difference of 10/71 and 10/70 as 0, 8, 30, 40.^{P}

In the following figure al-Maghribī discusses about triangles and compares them, then gives two values related to triangles.[65]

First of all two lines AB and BC are given, where AB>BC and they are perpendicular to each other. As a result of joining the points A and C, ABC becomes a right-angled triangle, which is circumscribed by a circle. DH is drawn, meet at H on AB and perpendicular to AB. D and C are joined and increased upto R which is a point at which the increased line AB meets.

65. Ibid.

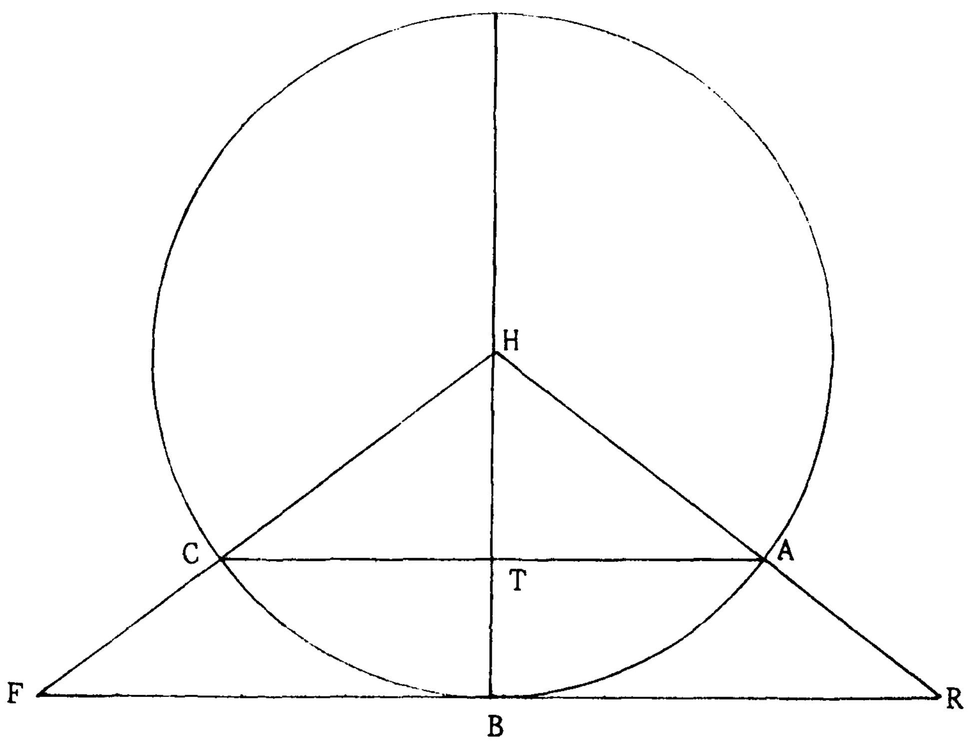

FIG. 5

AC (=2 AT) < arc ABC < RF

Sin AB (=3/4°) = AT = o; 47, 7, 21, 7, 37^P

Since ΔRFH $\sim$ Δ AHC, therefore [$\sim$ indicates similarity]

RF/AC = BH/TH

RF = 1; 34, 15, 11, 19, 25^P

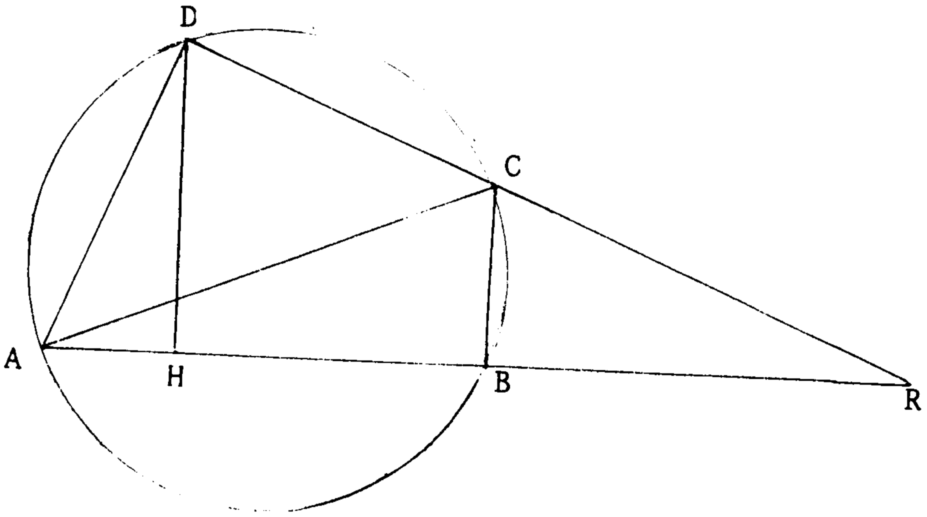

FIG. 6

From the above figure he deduced that

$HR = AB$; $RH/DH = BA/DH$

$AH = BR$; $RH/DH = DH/HA$, since $\angle D = 90^{o}$

But

$$RH/DH = RB\ (=HA)/BC$$

$$BA/DH = DH/HA = HA/BC$$

C H A P T E R - III

SPANISH-MUSLIM MATHEMATICIANS OF LATER MEDIEVAL PERIOD (13TH, 14TH AND 15TH CENTURIES A.D.)

Ibn Badr

Abū 'Abdullāh Muḥammad ibn 'Umar ibn Muḥammad also known as ibn Badr (Sp: Abenbeder) is reported to live in Seville. Unfortunately his exact flourishing period is not known but Sarton doubtfully mentions that he flourished "...probably in the thirteenth century".[1] No other biographical sketch of ibn Badr is available.

As far as his mathematical works are concerned, it is reported that he wrote on algebra and arithmetic. He discussed in his work, Ikhtiṣār al-Jabr wa'l-Muqābalah (Compendium of al-Jabr wa'l-Muqābalah of al-Khwārizmī), different branches of algebra and arithmetic such as numerical examples or problems, quadratic equations, Surds,[2] multiplication of polynomials, arithmetical

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1. G. Sarton, op. cit., Vol. II, Part II, p.622.

2. A Surd is defined as a numerical radical in which the radicand is rational but the radical itself is irrational.

theory of proportion and linear Diophantine equations.[3] One part of this work deals with theoretically also. This work of ibn Badr has some importance for a commentary on this work was written in verses in 1311-12 A.D. by Muḥammad ibn al-Qāsim al-Gharnāṭī.[4] It is an unavoidable fact, if all the works of Ibn Badr are found, including those of all the commentaries written by him or on his own works, there would be some more inventions and contributions to the field and those contributions might help the mathematicians of later period.

Muḥammad al-Ḥaṣṣār

Abū Zakariyā (or Abū Bakr) Muḥammad ibn 'Abdullāh al-Ḥaṣṣār was a mathematician of high calibre. The exact period of his life could not be determined. Sarton in this connection, bases Ibn al-Banna as his argument that the Talkhīṣ, a great work of mathematics of ibn al-Banna was an offshoot of his work and on this ground he says that al-Ḥaṣṣār flourished in 12th or 13th century A.D.[5] Smith, regarding his period mentions him as a 12th century mathematician.[6]

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3. Muḥammad Sa'ūd, Islam and Evolution of Science, p. 36
4. Ibid.
5. G. Sarton, op. cit., Vol. II, Part I, p.400.
6. D.E. Smith, op. cit., Vol. I, p. 210.

No more mathematical works are reported to be written by him except a treatise on arithmetic and algebra. This treatise has such an important work on arithmetic and algebra that ibn al-Banna, a renowned mathematician of Maghrib whose work Talkhīṣ exercised great influence on the Islamic East and the West, derived this work from the work of ibn al-Ḥaṣṣār.[7] Ibn al-Ḥaṣṣār, in this work uses Ghubār numerals. Later it was translated into Hebrew in 1271 or 1259 A.D.[8] by Moses ben Tibbon.

Ibn al-Raqqām (d. 1315 A.D.)

Abū 'Abdullāh, Muḥammad ibn Ibrāhīm Ibn al-Raqqām al-Awsī al-Mursī was such a scholar of Muslim Spain who worked on physics and astronomy alongwith mathematics. We are unable to get more information regarding his life and intellectual contributions. He was born in the thirteenth century A.D. It is expected due to his nisba, 'al-Mursī' that he was probably born at Murcia.[9] He died according to Brockelmann[10] and

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7. G.Sarton, op. cit.
8. D.E. Smith, op. cit.
9. G. Sarton, op. cit., Vol. III, Part I, p. 695.
10. C. Brockelmann, op. cit., Supt. Bond II, p.378.

Sarton [11], on May 27, 1315 A.D./Ṣafar 21, 715 A.H. in Granada, where he practiced as a physician for many years before his death.

No mathematical writing of ibn al-Raqqām is survived but it is said that he wrote on scientific instruments, mathematics, astronomical tables and other subjects one of which, Kitāb fī 'ilm al-Ẓilāl is still survived.[12]

Ibn Bāsa (d. 1316 - 17 A.D.)

Abū 'Alī al-Ḥasan ibn Muḥammad ibn Bāsa was a great maker of astronomical instruments, theologian, astronomer as well as a mathematician, flourished at Granada. His forefathers were Jews but embraced Islam in the Eastern Spain where they earlier lived. So he was born in a Muslim family of Jewish origin. In the great mosque of Granada he was appointed as 'mu'addil' (chief time computer). In religious affairs he bore a great responsibility there. He died there in Granada in 1316-17 A.D.[13]

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11. G. Sarton, op. cit.

12. Ibid., and C. Brockelmann, op. cit.

13. G. Sarton, op. cit., Vol. III, Part I, p.696.

No mathematical work by ibn Bāsa is reported to be written by any means, while Sarton shows doubt in his writing a treatise on the astrolabe al-Ṣafīḥah al-Jāmi'a li Jāmi'a al-'urūḍ which was written in 160 or 161 chapters and was completed in 1274 A.D.[14]

Ibn Bājja (d. 1339 A.D.)

Abū Bakr Muḥammad ibn Yaḥyā ibn Sā'igh ibn Tujībī ibn Bājja (son of goldsmith), popularly known in the Latin World as Avenpace or Avempace, was a great philosopher of Muslim Spain as well as Muslim world. He is also regarded as a renown musician, poet and composer of popular songs. He simultaneously studied mathematics, botany and astronomy.[15] He may also be included among scientists, physicians, and commentators of Aristotle in addition to philosophers.[16] He is said to have been born in the late 11th century A.D. at Saragossa and lived in many cities of Spain, particularly Seville and Granada and North Africa. He died in Fez, perhaps due to poisoning in May 1139 A.D./446-47 A.H.

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14. Ibid.

15. D.M. Donlop, E.I. (New Edition), Vol. III, pp. 738-39.

16. G. Sarton, op. cit., Vol. II, Part I, p. 183.

A number of works on mathematics, alchemy, medicine, astronomy, natural science and music (losted) are reported to be written by ibn Bājja. Some of the works which are attributed to him are : Risālah al-Wada' (letter of farewell); Risālah ittiṣāl al-'aql bī'l-insan (treatise of the union of the intellect with man); Kitāb fi'l-nafs (book on the soul); and the most important work Tadbīr al-mutawaḥḥid (rule of the solitary).[17]

As a mathematician, ibn Bājja wrote on geometry. It is uncertain whether it had some importance or not. However, infact he was a man of great intellect whose works were the landmark for the later scholars of different fields particularly philosophy. According Eḥsānul Huq Sulaimānī, "...the greatest work on Muslim philosophy by the famous scholar, Ibn Bājja is Tadbīr al-Mutawaḥḥida.[18]

Al-Qalaṣādī (1412 - 1486 A.D.)

Abū'l- Ḥasan 'Alī ibn Muḥammad ibn 'Alī al-Quraishī al-Bastī al-Qalaṣādī is among the last prominent Muslim mathematicians of Spain. He was born

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17. D.M. Donlop, op. cit., and G. Sarton op. cit.

18. Ehsanul Huq Sulaimani, Musalman Europe Main, p. 183.

at Bāzā (Bastā) in 1412 A.D. where he got his education in Law, Qur'an, exegesis, belles-letters and the science of the fixed shares (farā'iḍ) in an estate. The occupation of the city by Christians resulted his migration to Gharnāṭa (Granada) where he studied philology, science and philosophy under Abū Isḥāq Ibrāhīm bin Futūḥ and Muslim law under 'Adullāh al-Sarqusṭī. His interest for the sake of knowledge took him to North African as well as Egyptian cities where he studied science and belles-letters under renown scholars of the time. After his return from pilgrimage he devoted his life in writing and teaching at Granada.[19] Inspite of political disturbances in the country al-Qalaṣādī is credited to write a number of mathematical works as well as books on fixed shares, Mālikī law, Hadīth, apologetics relating to Prophet Muḥammad (peace be upon him), grammer, prosody etc. , some of which are considered to be great and got familiarity in both the East and the West.[20] He died in Bāja in Ifrīqiyā in 1486 A.D.

He is said to contribute different branches of arithmetic and algebra apart from other writings

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19. M. Suissi (I), E.I. (New Edn.), Vol. IV, p.476

20. Ibid.

attributed to him. He introduced many mathematical symbols[21] for the first time and excels other Muslim mathematicians of the East and the West.[22] Al-Qalaṣādī stands first to categorize algebra into three classes:[23]

1. 'Rhetorical algebras', having no symbol;
2. 'Syncopated algebras', written in words except abbreviations for certain operations and ideas; and,
3. 'Symbolic algebras', all operations of which are written in symbolism.

He worked on equations and found out the formula for finding the approximate square-root. Al-Qalaṣādī also paid his attention to the classification of fractions and on arithmetical procedures.[24]

The symbols which al-Qalaṣādī used are: 'j', in place of 'jadhr' (means root or square-root); 'Sh' for 'Shay' (thing) that represents the unknown 'x'; 'm', for 'mal' (x^2); 'k' for 'Ka'b' (x^3) and 'l' which is taken from the verb 'Ya'dilū' (for 'ta'dīl') to represent 'equal to' (=). Some other symbols, like 'Wa' for

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21. Short Arabic letter or words for denoting mathematical terms.

22. F. Cajori, op. cit., p. 111.

23. Ibid.

24. Ibid.

'addition', 'illā' for 'subtraction', fī' for multiplication and ' 'alā' for division were also used by him in his works.

Al-Qalaṣādī's writings on arithmetic and algebra looks to be same as the works of some scholars of different civilizations. For A.S. Saidan says:

> Like similar works from the thirteenth century on, al-Qalaṣādī's writings show Arabic arithmetic and algebra when their constituents-ancient manipolational tradition, Hindu techniques, and Greek number theory are combined to form one entity. But they also reflect a civilization on the wave, for most of them are commentaries, summeries or summeries of the works of al-Qalaṣādī himself or by others.

Al-Qalaṣādī was also familiar with the sequence Σn^2 and Σn^3 which he discussed in his writings and found the roots of imperfect squares with the help of successive approximations, that if r_1 is an approximation of $\sqrt{n}$ and $r_2 = n/r_1$ then $r_3 = \frac{1}{2}(r_1+r_2)$ is a better approximation. Though he was not first to use it but stands first to stress it.[25] Earlier sexagesimal fractions[26] were used by the Eastern mathematicians for the purpose. Similarly with the method of continued fractions, his approximation of $\sqrt{a^2 + b}$ is described as

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25. A.S. Saidan, <u>D.S.B.</u>, Vol. XI, p. 229.

26. These are the fractions introduced by the Babylonians

$(4a^3 + 3ab)/(4a^2 + b)$. The solution of the equation of first degree by the use of 'double false position'[27] is also ascribed to him. According to this, let $ax + b = 0$, and let m and n be two numbers, called 'double false position', further suppose $am + b = M$ and $an + b = N$, then al-Qalaṣādī derived as $(nM - mN) \div (M - N) = x$.

Keeping in view the above mentioned equations i.e. $ax + b = 0$, $am + B = M$ and $an + b = N$, the value of x may be proved by equalizing $(nM - mN) \div (M - N)$ with $-b/a$.

Take $\dfrac{nM - mN}{M-N}$

$$= \frac{n(am+b) - m(an+b)}{am + b - an - b} \quad [\text{Since } M = am + b \text{ and } N = an + b]$$

$$= \frac{anm + nb - anm - mb}{am - an}$$

$$= \frac{b(n-m)}{a(m-n)}$$

$$= \frac{-b(m-n)}{a(m-n)}$$

$$= -\frac{b}{a}$$

Al-Qalaṣādī wrote a book on algebra which is a commentary of <u>al-arjūzā al-Yāsmīniya</u> of ibn al-Yāsmīnī (d.1204 A.D.).[28] The <u>arjūzā</u> was written by ibn al-Yāsmīnī giving the rules of algebra in verses.[29]

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28. G. Sarton, <u>op. cit</u>., Vol. II, Part I, p. 400.

29. A.S. Saidan, <u>op. cit.</u>

Besides many commentaries and summaries of earlier works al-Qalaṣādī wrote a number of original works which start with the writing of al-Tabṣirah fī'ilm al-ḥisāb (classification of the science of arithmetic).

Following list of his arithmetical works is given by M. Suissi[30] and some names by C. Brockelmann [31].

1. Ghunyat dhawī'l-albāb fī sharḥ kashf al-jilbāb;
2. Qānūn al-ḥisāb wa ghunyat dhawi'l-albāb;
3. Inkishāf al-Jilbāb 'an funūn al-ḥisāb;
4. Kashf al-asrār (astār) 'an 'ilm (hurūf) al-Ghubār (unfolding the secrets of the use of dust letters), edited by F. Woepcke in his Traduction due traite d'arthmatique d'Abul Hascan 'Ali Ben Mohammad al Ḳalaṣādī and written during 1858 and 1859 A.D.[32] Kashf al-asrār was taught in some schools of North Africa for a long period;
5. Kashf al-Jilbāb 'an 'ilm al-hisāb (unveiling the science of arithmetic). This book was also edited by F. Woepcke in 1854.

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30. M. Suissi (I), op. cit.
31. Geschichte der Arabischan Literature.Supplement-band II, p. 378.
32. A.S. Saidan, op.cit.

6. Risālah fī ma'āni'l Kasr wa'l-Basṭ;
7. Risālah fī ma'rifat istikhrāj al-murakkab wa'l-basīṭ;
8. Sharḥ dhwāt al-asma'.
9. Sharḥ al-Talkhīṣ ibn al Banna or Talkhīṣ a'māl al-ḥisāb (Summary of arithmetical operations) of Ibn al-Banna;
10. Tabṣirh al-Mubtadī bi'l-Qalam al-hindī; and
11. Al-Tabṣirah al-wāḍiḥa fī masā'il al-a'dad al-Lā'i'ha.

A part from these works al-Qalaṣādī is attributed to many works on fixed shares[33] (farā'iḍ).

As far as al-Qalaṣādī's contribution to fraction is concerned, he stands first to use symbol and explained it with expressions in his own words.[34] The seperation of numerator and denominator with a line came into being first of all by the efforts of al-Qalaṣādī. He dealt the fractions in his book Kashf al-Asrār. He also explained the seperation by the terms ' 'alā ra'sihī' (means placed above it) and'mā fawq al-Khaṭṭa' (what which is above the line) in his commentary on Talkhīṣ of Ibn al-Banna. In this regard

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33 . M. Suissi (II) E.I., Vol. IV, p. 725.

34. Ibid.

he says "...one does not say four-fourth of five-fifth.[35] Al-Qalaṣādī described fractions of relationship (muntaṣib) and subdivided fractions or 'fractions of fractions' (Muba'ad). He gave the examples of these fractions, as of 'fractions of relationship' in modern notation,

that $\frac{3.1.4.5}{4.3.7.9}$ may be written as :

$$1 + \frac{3}{4}$$

$$4 + \overline{3}$$

$$5 + \overline{7}$$

$$\frac{\quad}{9} = \frac{5 \times 7 \times 3 \times 4 \quad 4 \times 3 \times 4 \quad 1 \times 4 \times 3}{9 \times 7 \times 3 \times 4}$$

$$= \frac{480}{756}$$

and fractions of fractions as :

$\frac{6\ 4\ 1}{7\ 5\ 3}$ denotes the multiplication of $\frac{6}{7}$, $\frac{4}{6}$ and $\frac{1}{3}$, which is requested as 24/106. In the later example, he multi-

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35. Ibid.

plies 6 with 4 and 1 and similarly 7 with 5 and 3 without using any symbol of multiplication.[36]

Like some other eminent mathematicians of Muslim Spain al-Qalaṣādī, alongwith his writings and researches produced many students of high rank who, on their turn produced their own works, contributed to the subject and got familiarity. Among them three, Abū 'Abdullāh al-Sanūsī (d. 1490 A.D.), Abū 'Abdullāh al-Mallānī and Aḥmad bin 'Alī al-Balawī studied mathematics and performed marvellous job.

Abū Abdullāh Muḥammad bin yūsuf al-Sanūsī who flourished at Tlemcen (North Africa) produced about twenty six books on mathematics, astronomy and Muslim lore. But no biographical sketch is available of other two scholars.

Al-Umawī (d. 1498 A.D.)

Abū 'Abdullah Ya'īsh ibn Ibrāhīm ibn Yūsuf ibn Simāk al-Umawī was also a great mathematician who threw light on the different sections of arithmetic and contributed them. He, like other Muslim mathematicians of the East worked on arithmetic and became master of the

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36. Ibid.

subject. His rank in the subject may be recognized by the fact that a number of students received knowledge from him some of whom belonged to as far distant place from Spain as Damascus. No more biographical sketch of al-Umawī is available except the reports of C. Brockelmann and A.S. Saidan that he died in 895 A.H./ 1489 A.D.[37] But the copyist, 'Abdul-Qādir al-Maqdisī who wrote a note on the ninth folio of his work on arithmetic, was given the licence to write on 17th Dhul Hijja 774 A.H. / 9th June 1373 A.D. . This shows that he lived in or before 1373 A.D.

Many books and treatises on arithmetic are attributed to al-Umawi.

1. Raf' al-ishkāl fī masāḥat ashkāl (Removal of doubts concerning the mensuration of figures) is a seperate treatise on mensuration in 17 folios.
2. R. fī'ilm al-Qabbān and
3. Marāsim al-intisāb fī'ilm al-ḥisāb.

The Marāsim, according to A.S. Saidan "...is significant

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37. C. Brockelmann, op. cit. Vol. II, p. 344, and A.S. Saīdan, op. cit.

in being written by a Western Muslim for the Easterners..."[38] He, in this work modifies the Indian method of calculation by 'dust board' which contains the method of counting by fingers and the pythagorean theory of numbers also. The trend used by al-Umawī exercised great influence in the West.

Several methods of multiplication, addition as well as subtraction were invented, and described their principal operation briefly by him while dealing with Indian arithmetic. Al-Umawī did not use the numerals except in the tables of sequences.

Al-Umawī's openion regarding fractions differed from the mathematicians of the East and also India who used to write it as $\begin{matrix}a\\b\end{matrix}$ or $\begin{matrix}0\\a\\b\end{matrix}$ and is credited to be among those, like at-Qalaṣādī (1412 - 1486 A.D.) who presented the concept of writing fraction as a/b using a line between the numerator and the denominator. In every operation, the numbers should be separated from the steps of operation by underlining it, described by him. It is remarkable that, though the Eastern mathematicians worked more on different branches of mathematics and their work is of high standard than that of the Western

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38. Ibid.

Muslims but at some place the Wester mathematicians presented such work that was unknown in the East. The later work of al-Umawī (seperation by a straight line) is of that kind which came to be known in the East late in the middle ages.[39]

The fields on which al-Umawī threw light and worked are the mensuration, square roots and cubic roots, sequences and series of polygonal and pyramidal numbers, arithmetical and geometrical progressions, number theory and others as summation of r^3, $(2r+1)^3$, $(2r)^3$, $r(r+1)$, $(2r+1)(2r+3)$, $2r(2r+2)$ from $r = 1$ to $r = n$.[40]

While dealing with addition, al-Umawī used the summation of sequences in which he followed the process of summation started by Babylonians and used by Diophantus as well as Arab mathematicians.[41] The trend was used by him by taking the sum of ten terms as an example in which he used no symbol. Like the work of Hindu mathematicians and Arab arithmetic in which nine and other numbers are considered to be casted out in subtraction he considers casting out[42] sevens, eights, nines and elevens.

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39. Ibid.

40. Ibid.

41. Ibid,

42. The act of obtaining the remainder when a given natural number is divided by an integer n , is known as <u>Castingout n's</u>.

The invention of the rule $N = a_o + a_1 \times 10 + a_2 \times 10^2 \ldots\ldots$

$$= \Sigma\, a_s \times 10^s$$

is attributed to al-Umawī where N is given as any integer in any decimal scale. Also if P is any other integer and after casting out P's from N the remainder of 10^s is r_s [i.e. $10^s = r_s \pmod{P}$], he found out $\Sigma a_s = r_s$ which is divisible both by P as well as N. The rule is lead by consideration of casting out sevens, eights and elevens. He, when takes 7 as the value of P, presents $r_s = (1, 3, 2, 6, 4, 5)$. Al-Umawī gave more attention to 'the sum of natural numbers', 'natural odd and even numbers' and in geometrical progression, 2r and $\sum_{r=0}^{n} 2r$

While dealing with square and cube roots, he gave rule of approximation. Though his rules on square and cube roots are not as developed as those of the Eastern mathematicians but infact he did not copy the rules of the East, as the method of extracting roots of higher order.[43] His rules for finding perfect square and cube, according to Saidan "...have not been found in other texts".[44] Al-Umawī discloses some rules with supposing 'n' as a perfect square.[45]

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43. Ibid., p. 540.

44. Ibid.

45. Ibid.

1. The unit digit of the number should be 1,4,5,6,9, or end with even zeros (i.e. Two zeros, 4 zeros ...)
2. Every tens' place must be odd if the unit place is 6 otherwise it should be even.
3. If the unit place of a number is 1 then the hundreds' place and half of the tens' place must be both even or both odd.
4. Similarly every number having unit place.
5. Must have tens' places
6. $n \equiv 0, 1, 2, 4 \pmod 7$

   $\equiv 0, 1, 4 \pmod 8$

   $\equiv 0, 1, 4, 7 \pmod 9$

Similarly while dealing with perfect cube n, he expressed the following:

1. If the numbers have their last digit as 0, 1,4, 5,6, or 9 then their cube roots end with 0, 1, 4, 5, 6 or 9 respectively. Also if the last digits are 3,7,2 or 8 then the roots end with 7,3,8 or 2 respecitvley and
2. $n \equiv 0, 1, 6 \pmod 7$

   $\equiv 0, 1, 3, 5, 7 \pmod 8$

   $\equiv 0, 1, 8 \pmod 9$

A separate treatise in 17 folios on the mensuration of figures, Raf' al-ishkāl fī maṣāliḥ al-ashkāl (removal of doubts concerning the mensuration of figures) is attributed to al-Umawī which shows his interest in the field of mensuration.[46]

In this way it may be said that al-Umawī devoted his intellectual activities to mathematics, some of whose works were unknown by the Eastern mathematicians while some others differed more with their works, either in rules or formulae.

46. Ibid.

CHAPTER - IV

OTHER SPANISH-MUSLIM MATHEMATICIANS

There are some other Spanish-Muslim mathematicians whose biographical sketch and works are, either not available or they are not much more. They are reported to be existed, flourished and worked there. The European scholars have not only acknowledged their knowledge and ability but also taught their works even till the sixteenth and seventeenth centuries A.D.They wrote commentaries of the works of Muslims of Spain, translated their works into different languages and summerized some of them.

Among such mathematicians are Abū'Ubādah Muslim bin Aḥmad, also known as Ṣāhab-e-Qibla due to his intelligence and knowledge. He is also credited to be among the earlier mathematicians of Spain.[1] He lived in Cardova and died there in 907 A.D. He worked and wrote on arithmetic and astronomy. 'Abd al-G͟hāfir ibn

1. 'Abd al-Qawī Ḍia, op. cit., p. 972

Muḥammad was a geometrician. His personality may be recognised with the report that he taught Maslama al-Majrīṭī (d. 1007 A.D.), a great mathematician and 'leader of mathematicians of his time' (Imām al-riyāḍiyyīn)[2] of Spain. Yaḥyā ibn Yaḥyā also known as ibn Samīna of Cardova, inspite of being engaged in getting the knowledge of ḥadīth (tradition), astronomy and poetry etc, devoted his valuable time to study the mathematical science . Abū Muslim ibn Khaldūn (d. 449 A.H./ 1057 A.D.) was a noble of Seville. Ibn Khaldūn mentions in his Muqaddimah this name as 'Umar ibn Khaldūn. According to ibn Khaldūn, Ibn abī Usaybia mentions him in his 'Uyūn al-Ambiyā fī al-Aṭibba that his full name was Abū Muslim 'Umar ibn Khaldūn.[3] Perhaps these two names are of the same person. He, according to ibn Khaldūn, "...was a well known mathematician of Spain.[4] Ḍia expresses his views about

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2. M. Mun'im Khaffāja, Qissat al-Adab fī'l Andalus, p. 160.

3. Ibn Khaldūn (Tr.) Muqaddimah, Tr. by Maulānā Sa'd Ḥasan Khān Yūsufī, p.16.

4. Ibn Khaldūn, op. cit.

him and says that he had a deep knowledge in philosophy, arithmetic, astronomy and medicine etc.[5] A number of scholars were associated with him and studied mathematics under his able guidance. Among his pupils was Muḥammad bin 'Umar ibn Barghauth who is said to have died in 449 A.H./1057 A.D.[6] Ibn Barghauth was also a well-known mathematician of his time. Abū'l-Ḥasan al-Zahrāwī was an expert of numbers and geometry. He wrote a book on geometry in the name al-Muqāblah'an Ṭarīq al-Burḥān.[7] Abū'l-Ḥasan Mukhtār al-Ra'īnī (died in Cardova in 435 A.H./1043 A.D.) was a mathematician and an expert in the fields of geometry and arithmetic. Another scholar, Muḥammad al-Laith (fl. 11th century A.D.) belonged to Saragossa, also had command on geometry as well as arithmetic. Besides his observations on the movements of stars, he secured great deal of knowledge of arithmetic and geometry. He worked on the trisection problem and devoted a considerable time to write on the construction of regular polygons of seven and nine sides.[8] He died in Valencia in 405 A.H./1014 A.D.[9]

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5. 'Abd al-Qawī Ḍia, op. cit.
6. M. 'Abdal-Mun'im Khaffāja, op. cit., p. 168.
7. Dā'irat al-Ma'ārif al-Sha'b, Vol. II, p. 197.
8. D.E. Smith, op. cit., p. 285.
9. M. 'Abd al-Mun'im Khaffāja, op. cit., p. 668.

Ibn al-Waqshī (flourished al Toledo) was also a geometrician. Muqtadir bin Hūd, a prince of Saragossa was an advanced mathematician and astronomer. He wrote a number of books in these fields which unfortunately are not available now.

In the period of Muwaḥḥidūn the study of geometry was very much common. This period produced a number of Muslim mathematicians. Yaḥyā ibn Huzail was a mathematician, particularly geometrician who flourished in the period of Banū Aḥmar[10].[11] Hishām bin Aḥmad studied geometry and logic. He was a native of Toledo and died there in 454 A.H./1062-63 A.D. Similarly, 'Abd al-Raḥmān Ismā'īl al-Uqlīdī was such a scholar who worked on logic and geometry. He belonged to a learned family as his father was among the nobles of Cardova, bearing a position in the Judiciary there. Only one of his works Ikhtiṣār al-Quṭub al-Thamāniyah al-Manṭaqiya is came to be known, which is a summary of eight books on logic written by Aristotle, in one. This book, unfortunately is not available.[12] Muḥammad bin 'Alī bin 'Ābid al-Anṣārī was a scholar who, with

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10. They were the rulers at Granada (1232-1492 A.D.) of Nāṣirīd dynasty which was founded by Muḥammad ibn Yūsuf ibn Naṣr (1232-1273 A.D.)

11. Da'irat al-Ma'ārif al-Sha'b, Vol.II, p.197.

12. M. 'Abd al-Mun'im Khaffāja, op. cit., p.160.

the study of mathematics rendered his intellectual services for tafsīr, literature, poetry, grammer, and history. So it may be said that he was a master of many subjects. He received his education in his native city, Fez (North Africa) but later migrated to Spain and settled at Granada. U.R. Kaḥḥālah writes that he, "...later became Andalusī (Spanish)".[13] He died in Granada. He made shorter version of Tafsīr al-Kashshāf of Zamakhsharī in the name Mukhtaṣar Tafsīr al-Kashshāf li Zamakhsharī.

Abū Isḥāq Nūr al-Dīn al-Bitrūjī (fourished in the early thirteenth century) though, was an astronomer but may be included in the list of mathematicians due to his work on planetary motion in which he worked on mathematics also.[14]

Abū'l-'Abbās 'Abd al-Salām al-Faraḍī, born, flourished and died at Cardova (922 -3 A.D.), was a mathematician who worked on arithmetic which, according to Smith, "...was of some note".[15] Muḥammad bin

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13. U.R. Kaḥḥālah, Op. cit., p. 20-21.
14. D.E. Smith, Op. cit., p. 210.
15. D.E. Smith, Op. cit., p. 192.

Ismā'īl (d. 331 A.H./943 A.D.), commonly known as Ḥakīm was such a mathematician who, besides his works on the subject, studied logic and grammer also.[16]

There were some such mathematicians in Muslim Spain who worked on different branches of mathematics especially arithmetic and geometry or one of them on one hand and on the other hand with the help of mathematical calculations wrote on inheritance (farā'iḍ). Among such mathematicians are: Abū Ghālib Ḥabbāb bin 'Ubādah, an arithmetician; Abū'l-Aṣbagh 'Īsā bin Aḥmad al-Wāsiṭī, a pupil of ibn Khaldūn flourished at Cardova and studied astronomy also; Ibn al-'Aṭṭār who was a pupil of ibn al-Ṣaffār, was a distinguished scholar of arithmetic and geometry.

Arithmetic and geometry were more common among the scholars of Muslim Spain than other branches of mathematics. There are so many mathematicians in Muslim Spain who worked, got expertised and some of them even taught these branches at their respective flourishing places and their periods. 'Abdullāh bin Muḥammad was regarded by the ruler, al-Mustanṣir

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16. Ibn Ṣā'id Andalusi, Op. cit., p. 64.

billāh due to his intellegency in mathematics, jurisprudence, grammer, chemistry etc.[17] Abū Bakr bin 'Īsā rendered his services in teaching geometry, arithmetic and astronomy. Abū'l-Qāsim Aḥmad bin Muḥammad al-'Adwī al-Ṭanbarī worked on geometry and arithmetic and wrote on Mu'āmlāt. Abū Marwān Sulaimān bin Muḥammad al-Nāsī was one of the pupils of ibn al-Samḥ. He came to be known as an expert of geometry and arithmetic. Another pupil of ibn al-Samḥ, Abū Ja'far Aḥmad bin 'Abdullāh also worked there as a mathematician. 'Abdullāh bin Aḥmad of Saragossa was a distinguished geometrician, arithmetician and astonomer who, as per tradition taught a number of students. With the reference of his pupil, 'Alī bin Dā'ūd, ibn Ṣā'id mentions, "...He did not see such high ranked mathematician as 'Abdullāh was". He died at Valencia in 448 A.H./1056 A.D. Similarly ibn Barghauth produced a number of Muslim mathematicians in Spain who, later on became masters of the subject and worked individually for the purpose of contribution to the subject. Three of his students, ibn al-Laith, ibn Ḥaī and ibn Jallāb were prominent. Ibn Ḥaī, al-Ḥasan

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17. Ibid. , p. 68.

bin Muḥammad bin al-Ḥusain al-Tajībī lived at Cardova and worked in the fields of geometry and arithmetic. He died in Yaman in 456 A.H./1046 A.D. Al-Ḥasan bin 'Abdul Raḥmān ibn Jallāb studied physics, logic, astronomy and geometry. Of the mathematicians of Toledo, the name Abū Ja'far Aḥmad bin Ḥumays bin 'Āmir bin Manīḥ (d. 454 A.H./1062 A.D.) may not be neglected who, at the same time was astronomer, physician and linguist.

CONCLUSIONS

Spain, inspite of existing in Europe and much far removed from Arabia, with internal and external distrubances due to Christian as well as Muslim interventions and coups, produced a number of Muslim mathematicians from the middle of the tenth century till the downfall of Muslim rule in 1492 A.D. Europe is regarded as the most advanced continent in intellectual development especially the field of education. This became possible due to the renaissance and later developments, which was ofcourse the direct result of a great deal of translations of Arabic works into European languages and the impact of the culture, civilization and the developments of Spain on it.

The advancement of Muslim Spain in almost every field of human life was higher than any other state of Europe. It was the most populous, richest and the most enlightened country. The people enjoyed facilities and luxuries. They contributed towards the development of its industries, agriculture, architecture and other fields. A marvellous beautification of the country was carried out through the construction of historical buildings, like palaces, mosques, aqueducts and bridges, and the maintenance of beautiful parks by the Spanish people. A large number of new industries of silk weaving, paper and glass-making and handicraft were established and its products were exported to many countries of Africa and even to India. They perfected the irrigation system, fertilized the land and imported many new trees, fruits and vegetables which till then were not known in the country. They became masters of many arts such as enamelling, demascening and mining which largely contributed towards the upliftment and prosperity of the country. When the country was developing at a rapid pace, its scholars were busy devoting their full energies in intellectual activities.

Their advancement in almost all sciences made Spain an intellectual centre which attracted students and scholars from all over Europe. Some of them studied in its colleges and Universities and others after learning the Arabic language engaged themselves in translating the works of Muslims and Greeks from Arabic into European languages, especially Latin and Hebrew. It is noteworthy that the Christians and Jews worked side by side in carrying out the translation work.

As the ninth century is regarded as the period of translation from Greek into Arabic so the twelfth century is regarded as the period of translation of Arabic works into European languages, although the process of transmission of Muslim sciences actually had already begun in the eleventh century itself. In the translation of scientific works more priority was given to astronomical, arithmetical, trigonometrical and geometrical works. Spain, in this regard played an important role, as most of the translation works were carried out either directly or indirectly through it. The Arabic works were so dominating that the European translators came to know all the Greek works only through their Arabic translations instead of their original texts.

Adelard of Bath, though he never visited Spain himself, is credited with the translation of Elements of Euclid into Latin which is regarded as the earliest translation and foundation of all editions translated till 1533 A.D. The astronomical tables of al-Khwārizmī and the Quran were also translated by him. His contemporary, Plato of Tivoli translated the astronomical works of al-Battānī and spherics of Theodosius. John of Seville translated a number of works written by the Arab authors. Gerard of Cremona (flourished in the second half of the twelfth century) translated the works of Spanish-Muslim mathematicians as well as of Greeks. Seventy two books are reported to be translated by him. They include Qānūn al Magisṭī of Ptolemy, the Spherica of Theodosius, fifteen books of Euclid, a work of Menelaus, De Numero Indico, an algebraic work of al-Khwārizmī, Iṣlāḥ al-Majisṭī of jābir ibn Aflaḥ, On the Dawn and Tabulae Jahen of al-Jayyānī and others. Abraham ben Ezra (1047 - 1169 A.D.) of Toledo was the famous and influencial translator from amongst the Jews. He is distinguished to explain the numeration system of the Arabs and its fundamental process in his arithmetic. Robert of Chester, an Englishman studied at Barcelona with Plato of Tivoli and translated algebra of al-Khwārizmī. Danial morley, another Englishman visited Toledo and quoted the writings of the Arabs while writing his own work on astronomy and mathematics. Ya'qūb (Jacob) ben Mahir was a prominent Jew who translated Iṣlāḥ al-Majisṭī of Jābir in 1274, Elements and Data of Eucid, and Sphere of Menelaus into Hebrew from Arabic. Mūsā (Moses) ben Tibbon translated many works into Hebrew including Jābir's Iṣlāḥ and the work of ibn al-Ḥaṣṣār (d. 1259 A.D.), the title of which is not known.

The other translators who either translated the Arabic works and worked on them were; Judah ben Mūsā (Moses) who translated <u>al-Bāri' fī Aḥkām al-Nujūm</u> of ibn Abī al-Rijāl into Castellian; Ismā'īl bin Yahūda (Semuel ben Jehuda) who translated <u>The Total Solar Eclipse</u> and <u>On the Dawn</u> of al-Jayyānī into Hebrew; Arnol of villanova and Judah Nathan who translated the work of al-Zarqālī; Richard of willingford used <u>Iṣlāḥ</u> in his <u>Alion</u> and <u>De Sectore</u> while Simon Bredom discussed <u>Iṣlāḥ</u> in his commentary on <u>Almagest</u>; Henery of Seville and Pedro Nunez both studied the work of Jābir and commented on it.

There were some more translators of Spain, France, Italy and England etc. who, in their respective periods separately translated various Arabic works and put their names in the list of translators of that period. The most prominent amongst them are Walcher of Malvern and Michael Scot of England; Hugh of Santalla was such a Spanish who worked in the field other than those who worked under the patronage of king Alfonso and Hermann of Dalmatia; Rudolf of Bruges and Henry Bate belonged to Flanders; Armengand and Ya'qūb (Jacob) Anatoli were French; and Aristippus Catenia, Salio of Padua and John of Brescia were Italians.

The study of mathematics, as discussed earlier, was started and developed since or before 2700 B.C. when Sargeon, the ruler of Babylonia flourished and showed his interest in mathematics. Since then new contributions and additions were made in this field continuously inspite of the decay of various contributing civilizations, nations, races and schools. Different branches were developed and new theorems were derived according to the need of the people of the time. Almost every civilization had in use its own numerals. If the period of Muslim Spain is compared with that of the other civilizations, it would be quite obvious that it was shorter and more disturbed than the others, but their contributions were marvellous. Like other civilizations, spanish-Muslim mathematicians used their own numeration system by the name 'Dust Numerals' (Ḥurūf al-Ghubār). Commercial arithmetic in the form of complete science was introduced by al-Majrīṭī. Many works were also written on this science. Al-Jayyānī determined the magnitudes of the arcs on the surface of a sphere. Jābir was a prominent mathematician who worked on spherical trigonometry more than any other branch. He is regarded as the first to introduce the formula of right angled spherical triangle (Cos A = Cos a Sin B) and is credited to solve the problems of plane trigonometry with the help of whole chord. Ibn al-Yāsmīnī seems to be the only one in Spain who wrote his algebra in verses. Muḥyī al-Dīn al-Maghribī wrote a number of books, revised Greek works, derived a number of new geometrical theorems and at the same time proved them. He had given some thoughts towards trigonometry also. Ibn Badr discussed in his compendium a great deal of mathematics. Al-Qalaṣādī is credited to be a revolutionary man in this field who introduced new arithmetical and algebraic symbols, categorized algebra, worked on square root, classified fractions, gained familiarity with sequences, derived new formulae and wrote a number of books on different subjects especially mathematics. Al-Umawī, with the help of his writings on different sections of arithmetic exercised great influence on both the East and the West. He paid more attention on sequences and series and described many rules regarding perfect squares and cubes.

Apart from the mathematicians discussed above there is a long list of other Muslim mathematicians who existed in Spain, most of whom studied and worked on arithmetic and geometry and commanded great respect in this field both in the East as well as the west.

It is an established fact that traditionally, most of the scholars of Muslim Spain did not confined themselves to any particular field. They, apart from their own subjects, tried to get acquainted with other subjects also and worked on them. This effort shows their courage and hard work which continued inspite of the hostality of the Christians towards the Muslims and their sciences. It was mainly due to the untiring efforts of such scholars that politically and academically Spain, in due course of time, became a treasure of sciences especially that of mathematics.

If their works are further searched in different libraries (private as well as official), academies and other places in Spain, Maghrib (Morocco and Northern part of Algeria), and France, which played an important role in the politics of Spain, a huge material on mathematics and a number of new mathematicians may likely be discovered who, unfortunately, are unknown today.

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APPENDIX - 1

Ikhwān al-Ṣafa' was a group of scholars, most probably of Ismā'īlīs, lived at Basra who formed a high ranked brotherhood among themselves. Their purpose of forming this group was to awake the people from the 'dream of negligence' through spiritual education and training. This group produced fiftytwo Rasā'il (treatises) on various subjects which they divided into three categories and called as Rasā'il Ikhwān al-Ṣafa' wa Khillān al-Wafa.

I. The primary sciences (riyādiyah)

II. Religious sciences (al-Sharī'at al-Wad'iyah)

III. Philosophical sciences (al-falsafat al-Haqīqiyah)

The Rasā'il of Ikhwān al-Ṣafa' about which Y. Marquet writes in E.I. (Vol. III, P.1072) with the reference of al-Majrīṭī (d. 398/1007) that they were approximately composed between 350/961 and 375/986. They are as follows:

I. Mathematical and Educational Treatises:

1. Properties of numbers
2. Geometry
3. Astronomy
4. Geography

5. Music
6. Educational values of these subjects
7-8. Various scientific disciplines.
9. Actions and sayings of the Prophets and Sages.
10-14 Logic (including the Isagoge, the ten categories, peirhermenias, prior to, posterior analytics).

II. **Science and Natural Bodies :**

1. Explanation of the notions of matter, form, movements, time, space, and so forth.
2. The sky and the Universe.
3. Heneration and corruption
4. Meteorology
5. Formation of minerals
6. Essence of nature
7. Species of plants
8. Composition of human body
9. Explanation of the generation of animals and their species
10. Preception of the senses and their object
11. Embryology
12. Man as a microcosm

13. Development of particular souls in the human body
14. Limits of human knowledge and science
15. Maxims of life and death
16. Characters pertaining to pleasure
17. Causes of the diversity of languages, their system of transcription and calligraphy

III. Psychological and Rational Sciences:

1. Intellectual principles according to Pythagoras
2. Intellectual principles according to the Ikhwān
3. That the Universe is a macrocosm
4. Intelligence and the intelligible
5. Periods and the epochs
6. Essence of passion
7. Resurrection
8. Species of movement
9. Cause and effect
10. Definitions and descriptions

IV. Theological Sciences on the Nāmūs and the Sharī'ah :

1. Doctrines and religions

2. The character of the path leading to God
3. Explanation of the doctrine of the Ikhwān
4. Ways of life of the Ikhwān
5. Essence of faith and the virtues of the believing initiates
6. Essence of the Divine namus, conditions and virtues of the Prophets.
7. The manner of appealing to God
8. State of spiritual beings
9. Politics
10. Hierarchy inherent in the universe
11. Magic and talisman

Following are the letters and their respective values used by Ikhwan al-Safa'

a	b	j	d	h	w	z	ḥ	ṭ	i,y	k	l	m
1	2	3	4	5	6	7	8	9	10	20	30	40

n	s	'	f	ṣ	q	r	sh	t	th	kh	dh	ḍ
50	60	70	80	90	100	200	300	400	500	600	700	800

ẓ	gh	bgh	jgh	...
900	1000	2000	3000	

APPENDIX - 2

Double False Position :

The rule of 'double false position' is reported to be introduced by the Indian mathematicians. This rule was also used by the Arab mathematicians in their writings. It is the oldest method for approximating the real roots of an equation. According to this rule, let x_1 and x_2 be two numbers lying close to and on each side of a root x of the equation $f(x) = 0$. Then the intersection with the x-axis of the chord joining the points $(x_1.f(x_1))$, $(x_2.f(x_2))$ gives an approximation x_3 to the sought root.